Two-sample Instrumental Variable Regressions with Potentially Weak Instruments

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Abstract. We develop a Stata package for two-sample instrumental variables regression models with one endogenous regressor and potentially weak instruments. The package includes the classic two-sample 2SLS estimator whose inference is only valid under the assumption of strong instruments, as well as statistical tests and confidence sets with the correct size and coverage probabilities even when the instruments are weak.

Keywords: st0001, two-sample 2SLS, weak IV, inference

1 Introduction

Conventional instrumental variables (IV) regression requires that the dependent variable, the endogenous regressor, and the instruments come from the same dataset; but in many cases, researchers can only observe the dependent variable and the endogenous regressor in two separate data samples (see Björklund and Jäntti 1997; Miguel 2005; Feldman 2010; Brunner et al. 2012; Siminski 2013; Olivetti and Paserman 2015, among many others). In such situations, a two-sample two-stage least squares (TS2SLS) estimation strategy is frequently adopted. The estimator was proposed by Angrist and Krueger (1992, 1995). Inoue and Solon (2010) developed valid inference under the assumption of strong first stage correlation between the endogenous variable and the instruments. When the first stage is weak, however, the TS2SLS estimation and inference strategy are not valid. Firstly, the estimator is no longer asymptotically normally distributed, invalidating classic Inoue and Solon (2010) inference. Secondly, the TS2SLS estimator may be subject to a large bias under a weak first stage, as the classic attenuation bias of the estimator is inversely proportional to the strength of instruments.

In a recent study, Choi et al. (2018) developed weak-instrument robust tests and confidence sets for the two-sample IV regression model. This article develops a Stata package companion to this newly proposed inference method. Specifically, the new method extends the classic Anderson-Rubin (AR, c.f. Dufour 1997; Staiger and Stock 1997; Dufour and Jasiak 2001), Kleibergen (K, c.f. Kleibergen 2002), and conditional likelihood ratio (CLR, c.f. Moreira 2003; Andrews et al. 2006, 2008; Moreira 2009) tests and confidence sets to the two-sample IV regression model. We also consider cases of homoskedasticity and heteroskedasticity. The proposed package also reports the classic TS2SLS estimator with Inoue and Solon (2010) standard errors for completeness.

2 Model and Theory

Let subscript j, $j = 1, 2$ denote random variables in the first and second dataset with sample size n_j. Assume that $n_1/n_2 \to \tau$ for some fixed $\tau > 0$. In this article we consider the following two-sample IV regression model with independent and identically distributed data:

$$\begin{aligned} y_1 &= w_1\beta + X_1\gamma + \epsilon_1, \\ w_j &= Z_j\pi + X_j\psi + \varepsilon_j, \quad j = 1, 2. \end{aligned} \tag{1}$$

where y_1, w_1, ϵ_1 and ε_1 are $n_1 \times 1$; w_2, ε_2 are $n_2 \times 1$; Z_j is $n_j \times k$ for $j = 1, 2$; and X_j is $n_j \times p$ for $j = 1, 2$. All variables in the above model are observed except for w_1, the endogenous regressor in the first dataset. Without loss of generality, assume that $Z_j' X_j = 0$ for $j = 1, 2$. The orthogonality assumption is without loss of generality because one can always define new instruments as residuals from the regression of original instruments on exogenous regressors. We are interested in the inference problem of the parameter β in the outcome equation.

In the following, we discuss both the benchmark and the robust inference methods discussed in Choi et al. (2018). The benchmark method is developed under the same set of assumptions as in Angrist and Krueger (1992, 1995) and Inoue and Solon (2010), except for allowing for weak instruments. It, therefore, requires both homoskedasticity and equal moment of exogenous regressors in the two-sample IV model. The robust inference method, on the other hand, relaxes these two restrictions. In empirical applications, researchers might want to adopt the robust inference method for its generality and the benchmark method for a direct comparison with the TS2SLS results.

2.1 Benchmark Tests and Confidence Sets

In this section, we assume that the error terms ϵ_1, ε_1 and ε_2 in model (1) are homoskedastic, and the exogenous covariates X_1 and Z_1 in the first dataset have the same first and second moments as X_2 and Z_2 in the second dataset. We consider the weak IV asymptotic where the first stage parameter π in model (1) satisfies that

$$\pi = C/\sqrt{n_1} \text{ for some non-stochastic } k\text{-vector } C. \tag{2}$$

Let $\hat{\pi}$ and $\hat{\psi}$ be the OLS estimators from first stage regression of w_2 on Z_2 and X_2, and let $\hat{w}_1 = Z_1\hat{\pi}+X_1\hat{\psi}$ be the predicted endogenous regressor for the first dataset. Let $Y_1 = [y_1 \;\; \hat{w}_1]$, $a = [\beta \;\; 1]'$, $\eta = [\gamma \;\; \psi]$, and $V_1 = [u_1 \;\; v_1]$, where $\gamma = \gamma_1 + \psi\beta$, $u_1 = \epsilon_1 + \beta\varepsilon_1$, and $v_1 = Z_1(\hat{\pi} - \pi) + X_1(\hat{\psi} - \psi)$. The simultaneous equation model in equation (1) could be rewritten as

$$Y_1 = Z_1\pi a' + X_1\eta + V_1.$$

First, consider the two-sided null hypothesis $H_0 : \beta = \beta_0$ with some pre-determined significance level α. Let $b_0 = [1 \;\; -\beta_0]'$, $a_0 = [\beta_0 \;\; 1]'$. For any matrix X, we denote $X(X'X)^{-1}X'$ as P_X and define $M_X = I - P_X$. Define statistics

$$\hat{S}_n = (Z_1'Z_1)^{-1/2}Z_1'Y_1b_0/(b_0'\hat{\Omega}b_0)^{1/2}, \quad \hat{T}_n = (Z_1'Z_1)^{-1/2}Z_1'Y_1\hat{\Omega}^{-1}a_0/(a_0'\hat{\Omega}^{-1}a_0)^{1/2},$$
$$\hat{Q}_S = \hat{S}_n'\hat{S}_n, \quad \hat{Q}_T = \hat{T}_n'\hat{T}_n, \quad \hat{Q}_{ST} = \hat{S}_n'\hat{T}_n, \quad \hat{Q} = \begin{pmatrix} \hat{Q}_S & \hat{Q}_{ST} \\ \hat{Q}_{ST} & \hat{Q}_T \end{pmatrix},$$

where $\hat{\Omega} = \begin{pmatrix} \hat{\sigma}^2_{u_1} & 0 \\ 0 & \hat{\sigma}^2_{\varepsilon_2} n_1/n_2. \end{pmatrix}$ with $\hat{\sigma}^2_{u_1} = y_1'M_{[Z_1:X_1]}y_1/(n_1 - k - p)$ and $\hat{\sigma}^2_{\varepsilon_2} = w_2'M_{[Z_2:X_2]}w_2/(n_2 - k - p)$ are consistent estimators of $\sigma^2_{u_1}$ and $\sigma^2_{\varepsilon_2}$, the variance terms of u_1 and ε_2, respectively.

Next, define test statistics

$$\mathcal{T}_1(\beta_0) = \hat{Q}_S, \quad \mathcal{T}_2(\beta_0) = \hat{Q}^2_{ST}/\hat{Q}_T,$$
$$\mathcal{T}_3(\beta_0) = \frac{1}{2}\left[\hat{Q}_S - \hat{Q}_T + \left[\left(\hat{Q}_S + \hat{Q}_T\right)^2 - 4\left(\hat{Q}_S\hat{Q}_T - \hat{Q}^2_{ST}\right)\right]^{1/2}\right].$$

Under the weak IV asymptotic in (2), one can show that in the limit $\mathcal{T}_1(\beta_0)$ follows a χ^2 distribution with k degrees of freedom and $\mathcal{T}_2(\beta_0)$ follows a $\chi^2(1)$ distribution when the null condition $\beta = \beta_0$ holds. When $k = 1$, $\mathcal{T}_3(\beta_0)$ reduces to $\mathcal{T}_1(\beta_0)$. When $k \geq 2$, in the limit the probability of $\mathcal{T}_3(\beta_0)$ exceeding m is

$$p(m; q_T) = 1 - 2K\int_0^1 P\left[\chi^2_k < \frac{q_T + m}{1 + q_T s_2^2/m}\right](1 - s_2^2)^{(k-3)/2}ds_2$$

under the null, where $K = \Gamma(k/2)/\left[\pi^{1/2}\Gamma((k-1)/2)\right]$ and χ^2_k is a random variable following a χ^2 distribution with k degrees of freedom (Andrews et al. 2007).

Let $q_{1-\alpha}(k)$ be the $(1-\alpha)$ quantile of the $\chi^2(k)$ distribution. Define the decision rules of the three statistics as "reject the null if $\mathcal{T}_1(\beta_0) > q_{1-\alpha}(k)$", "reject the null if $\mathcal{T}_2(\beta_0) > q_{1-\alpha}(1)$", and "reject the null if $\mathcal{T}_3(\beta_0) > q_{1-\alpha}(1)$ when $k=1$, and reject the null if $p\left(\mathcal{T}_3(\beta_0); \hat{Q}_T\right) < \alpha$ when $k \geq 2$", respectively. All three tests have asymptotic size control under the weak IV asymptotic. When the null hypothesis $H_0 : \beta = \beta_0$ is violated, all three tests have nontrivial power dependent on the value of C. When the instruments are strong, all three tests have power approaching one. We call the test based on $\mathcal{T}_1(\beta_0)$ the TSAR test, the one based on $\mathcal{T}_2(\beta_0)$ the TSK test, and the one based on $\mathcal{T}_3(\beta_0)$ the TSCLR test.

Note that when $k = 1$, all three tests give identical results. When $k \geq 2$, TSCLR generally has better power performances than the other two methods, but there are also some data generating processes where TSAR can outperform. See Choi et al. (2018) for more details.

Given the proposed tests, the $(1-\alpha) \times 100\%$ confidence sets for β can be obtained by inverting the corresponding tests. Define

$$\mathcal{CI}_1(\alpha) = \{\beta_0 : \mathcal{T}_1(\beta_0) \leq q_{1-\alpha}(k)\}, \quad \mathcal{CI}_2(\alpha) = \{\beta_0 : \mathcal{T}_2(\beta_0) \leq q_{1-\alpha}(1)\},$$
$$\mathcal{CI}_3(\alpha) = \{\beta_0 : p\left(\mathcal{T}_3(\beta_0); \hat{Q}_T(\beta_0)\right) \geq \alpha\}.$$

The confidence sets, since inverted from asymptotic valid tests under the weak IV asymptotics, have correct coverage in the limit. When the instruments are weak, the confidence sets could be unbounded, which is an essential property for confidence sets to have correct coverage with arbitrarily weak instruments (Dufour 1997). The benchmark confidence sets are computed analytically following the fast computation method proposed by Mikusheva and Poi (2006) for the classic one-sample AR, K and CLR confidence sets.

Similar to the classic one-sample case (see Mikusheva and Poi 2006), the TSK method is generally not recommended in practice. This is because, for statistical testing, TSK has irregular non-monotonic power curve when $k \geq 2$, resulting in power loss with some data generating processes. For confidence sets, the TSK confidence set can also take the form of a union of two finite intervals, i.e., $[x_1, x_2] \cup [x_3, x_4]$, while the TSAR and TSCLR confidence sets, conditional on boundedness, only take the usual form of finite interval, or $[x_1, x_2]$ (See Table 1 below).

2.2 Robust Tests and Confidence Sets

This section relaxes the assumption of homoskedastic and equal moment of exogenous covariates, but the data is still assumed to be i.i.d. Specifically, we assume that Σ_{z,u_1} and Σ_{z,ε_2} are probability limits of $V[Z_1' u_1/\sqrt{n_1}]$ and $V[Z_2'\varepsilon_2/\sqrt{n_2}]$, respectively, and that $\Sigma_{l,ZZ}$ is the probability limit of $Z_l' Z_l/n_l$ for $l = 1, 2$, where it is not necessary for $\Sigma_{1,ZZ}$ and $\Sigma_{2,ZZ}$ to be equal.

Replace the first equation in model (1) by its reduced form

$$y_1 = Z_1\zeta + X_1\gamma + u_1.$$

Let $\delta = [\zeta \quad \pi]'$, we know that $r(\delta, \beta) = \zeta - \pi\beta = 0$. Let $\hat{\zeta} = (Z_1'Z_1)^{-1}Z_1'y_1$, $\hat{\pi} = (Z_2'Z_2)^{-1}Z_2'w_2$, and $\hat{\delta} = [\hat{\zeta} \quad \hat{\pi}]'$. It is easy to see that, for any β_0,

$$\sqrt{n_1}\left(r(\hat{\delta}, \beta_0) - r(\delta, \beta_0)\right) \Rightarrow N\left(0, \Sigma_{\beta_0}\right),$$

where $\Sigma_{\beta_0} = \Sigma_{1,ZZ}^{-1}\Sigma_{z,u_1}\Sigma_{1,ZZ}^{-1} + \tau\beta_0^2\Sigma_{2,ZZ}^{-1}\Sigma_{z,\varepsilon_2}\Sigma_{2,ZZ}^{-1}$ is a $k \times k$ variance-covariance matrix. Let $\hat{\Sigma}_{\zeta,\beta_0} = \frac{n_1^2}{n_1-k-p}(Z_1'Z_1)^{-1}\left(\sum_{i=1}^{n_1}\hat{u}_{1i}^2 Z_{1i}Z_{1i}'\right)(Z_1'Z_1)^{-1}$ and $\hat{\Sigma}_{\pi,\beta_0} = \frac{n_2^2}{n_2-k-p}(Z_2'Z_2)^{-1}\left(\sum_{i=1}^{n_2}\hat{\varepsilon}_{2i}^2 Z_{2i}Z_{2i}'\right)(Z_2'Z_2)^{-1}$, where $\hat{u}_{1i}$ is the i-th entry of $M_{[Z_1:X_1]}y_1$ and $\hat{\varepsilon}_{2i}$ is the i-th entry of $M_{[Z_2:X_2]}w_2$. Then Σ_{β_0} could be consistently estimated by $\hat{\Sigma}_{\beta_0} = \hat{\Sigma}_{\zeta,\beta_0} + \frac{n_1}{n_2}\beta_0^2\hat{\Sigma}_{\pi,\beta_0}$.

Following Magnusson (2010), the robust TSAR, TSK, and TSCLR test statistics for $H_0 : \beta = \beta_0$ can be written as,

$$\mathcal{T}_{1,robust}(\beta_0) = n_1(\hat{\zeta} - \hat{\pi}\beta_0)'\hat{\Sigma}_{\beta_0}^{-1}(\hat{\zeta} - \hat{\pi}\beta_0),$$

$$\mathcal{T}_{2,robust}(\beta_0) = n_1 \left[\hat{\Sigma}_{\beta_0}^{-1/2}(\hat{\zeta} - \hat{\pi}\beta_0)\right]' P_{\hat{\Sigma}_{\beta_0}^{-1/2}\hat{D}_{\beta_0}} \left[\hat{\Sigma}_{\beta_0}^{-1/2}(\hat{\zeta} - \hat{\pi}\beta_0)\right],$$

$$\mathcal{T}_{3,robust}(\beta_0) = \frac{1}{2}\left[\mathcal{T}_{1,robust} - \hat{q}_{\beta_0} + \left[(\mathcal{T}_{1,robust} + \hat{q}_{\beta_0})^2 - 4\left(\mathcal{T}_{1,robust}\hat{q}_{\beta_0} - \mathcal{T}_{2,robust}\hat{q}_{\beta_0}\right)\right]^{1/2}\right]$$

where $-\hat{D}_{\beta_0} = \hat{\pi} + \frac{n_1}{n_2}\beta_0\hat{\Sigma}_{\pi,\beta_0}\hat{\Sigma}_{\beta_0}^{-1}(\hat{\zeta} - \hat{\pi}\beta_0)$, and $\hat{q}_{\beta_0} = n_1\hat{D}'_{\beta_0}(\frac{n_1}{n_2}\hat{\Sigma}_{\pi,\beta_0} - (\frac{n_1}{n_2})^2\beta_0^2\hat{\Sigma}_{\pi,\beta_0}\hat{\Sigma}_{\beta_0}^{-1}\hat{\Sigma}_{\pi,\beta_0})^{-1}\hat{D}_{\beta_0}$. Under the null hypothesis, $\mathcal{T}_{1,robust}(\beta_0)$ and $\mathcal{T}_{2,robust}(\beta_0)$ have limit distributions $\chi^2(k)$ and $\chi^2(1)$, respectively, and $\mathcal{T}_{3,robust}(\beta_0) \Rightarrow \frac{1}{2}\left[\chi^2(1) + \chi^2(k-1) - q_{\beta_0} + \left[\left(\chi^2(1) + \chi^2(k-1) + q_{\beta_0}\right)^2 - 4\chi^2(k-1)q_{\beta_0}\right]^{1/2}\right]$, where $\chi^2(1)$ and $\chi^2(k-1)$ are independent chi-squared distributed random variables with 1 and $k-1$ degrees of freedom, respectively, given that $\hat{q}_{\beta_0} = q_{\beta_0}$. Therefore, we reject the null if so we reject the null if $\mathcal{T}_{1,robust}(\beta_0)$ or $\mathcal{T}_{2,robust}(\beta_0)$ is larger than $q_{1-\alpha}(k)$ or $q_{1-\alpha}(1)$, and if $p(\mathcal{T}_{3,robust}(\beta_0); \hat{q}_{\beta_0})$ is smaller than α, where $p(.,.)$ is defined in Section 2.1.

Similar to the benchmark case, robust TSAR, TSK, and TSCLR confidence sets of β can be constructed by inverting the robust TSAR, TSK, and TSCLR tests. The robust TSAR, TSK, and TSCLR confidence sets are computed using grid search. When the robust variance-covariance matrix $\hat{\Sigma}_{\beta_0}$ is replaced with $\bar{\Sigma}_{\beta_0} = \hat{\sigma}_{u_1}^2(\frac{Z_1'Z_1}{n_1})^{-1} + \frac{n_1}{n_2}\beta_0^2(\frac{Z_1'Z_1}{n_1})^{-1}\hat{\sigma}_{\epsilon_2}^2$, the three robust test statistics reduce to the benchmark counterparts.

3 Stata Implementation

By default, the **weaktsiv** command calculates the benchmark TSAR, TSK, and TSCLR tests and confidence regions discussed in Section 2.1. If the **robust** option is specified, **weaktsiv** calculates the robust versions of the tests and confidence regions discussed in Section 2.2. The command also reports the TS2SLS estimator with classic Inoue and Solon (2010) strong IV standard errors.

3.1 Syntax

`weaktsiv` *depvar varlist_exog (varlist_endog =varlist_iv)* [*if*] [*in*] [, `noconstant robust level(#)`
`test(#) point(#) grid(#(#)#)`]

depvar is the outcome variable.
varlist_exog is the list of exogenous variables.
varlist_endog is the endogenous regressor of the model.
varlist_iv is the list of exogenous variables used togehter with *varlist_exog* as instruments for *varlist_endog*.

3.2 Options

noconstant suppresses the constant term in the regression model.
robust provides the robust version of the two-sample weak IV tests that allow for heteroskedasticity and unequal moments of exogenous variables across the two samples.
level(#) sets confidence level; default is *level(95)*.
test(#) sets the hypothesized value of the endogenous variable's coefficient; default is *test(0)*.
grid specifies the grid used for confidence region calculation. *grid* could only be used together with the *robust* option because the benchmark confidence regions are calculated analytically. If not specified, the default uses the TS2SLS estimator plus/minus two times the Inoue and Solon (2010) standard error and 100 grid points.
point specifies the number of points used to create the grid for confidence region calculation. *point* could only be used together with the *robust* option, and could not be used together with the *grid* option. The default uses 100 grid points.

3.3 Saved Results

weaktsiv saves the following in e():

Scalars

`e(p_TSAR)`	TSAR test p-value	`e(p_TSK)`	TSK test p-value
`e(p_TSCLR)`	TSCLR test p-value	`e(TSAR_xi)`	endpoints of benchmark TSAR confidence sets
`e(TSK_xi)`	endpoints of benchmark TSK confidence sets	`e(TSCLR_xi)`	endpoints of benchmark TSCLR confidence sets
`e(level)`	confidence level for weak-iv robust inference	`e(H0_b)`	value of β under null for weak-iv robust inference
`e(n1)`	# of obs in the sample 1 (the outcome sample)	`e(n2)`	# of obs in the sample 2 (the endog. var. sample)
`e(chi2)`	TS2SLS Wald statistic	`e(F_first)`	TS2SLS first-stage F
`e(numinst)`	number of instruments	`e(df_m_first)`	TS2SLS first-stage residual degrees of freedom
`e(df_m)`	TS2SLS model degrees of freedom	`e(df_r)`	TS2SLS residual degrees of freedom
`e(r2)`	R^2	`e(r2a)`	adjusted R^2
`e(mss)`	TS2SLS model sum of squares	`e(rss)`	TS2SLS residual sum of squares
`e(rmse)`	TS2SLS root mean squared errors	`e(points)`	# of grid points for robust confidence sets

Macros

`e(cmd)`	`weaktsiv`	`e(robust)`	whether robust inference methods are used
`e(TSAR_type)`	type of benchmark TSAR confidence set	`e(TSK_type)`	type of benchmark TSK confidence set
`e(TSCLR_type)`	type of benchmark TSCLR confidence set	`e(TSAR_cset)`	robust TSAR confidence set
`e(TSK_cset)`	robust TSK confidence set	`e(TSCLR_cset)`	robust TSK confidence set
`e(grid)`	grid range for robust confidence set	`e(cons)`	whether constants are used
`e(instd)`	instrumented variable	`e(inst)`	instruments
`e(exog)`	exogenous variables	`e(depvar)`	dependent variable

Matrices

`e(b)`	coefficient vector	`e(V)`	variance-covariance matrix of the estimators

Functions

`e(sample)`	marks estimation sample

For the benchmark methods, the result types (i.e. `e(TSAR_type)`, `e(TSK_type)`, and `e(TSCLR_type)`) and endpoints (i.e. `e(TSAR_xi)`, `e(TSK_xi)`, and `e(TSCLR_xi)`) information could be used together to retrieve the exact confidence sets using Table 1.

Table 1: Benchmark TSCLR, TSAR, TSK Confidence Sets (Analytical Solution)

Test	Result Type	Interval
TSCLR	1	Empty set
	2	[x1, x2]
	3	(-infty, +infty)
	4	(-infty, x1] U [x2, infty)
TSAR	1	Empty set
	2	[x1, x2]
	3	(-infty, +infty)
	4	(-infty, x1] U [x2, infty)
TSK	1	Not used (not possible)
	2	[x1, x2]
	3	(-infty, +infty)
	4	(-infty, x1] U [x2, infty)
	5	(-infty, x1] U [x2, x3] U [x4, infty)
	6	[x1, x2] U [x3, x4]

4 Example

We use the dataset of Currie and Yelowitz (2000) and illustrate implementing the **weaktsiv** command in the just identified case. We estimate the effects of public housing on monthly rental payments in equations (2) and (3) in their paper. The following command illustrates the just identified case. The first half of the output reports the classic two-sample 2SLS estimation results with Inoue and Solon (2010) standard errors. The inference here is valid only when the instruments are strong. The second half of the output reports results from the weak IV robust tests and confidence regions discussed in Section 2.1. Since the model is just-identified here, only TSCLR is reported because TSAR, TSK, and TSCLR are equivalent under just-identification. The results are also reported in column (1) of Table 3 in Choi et al. (2018).

```
. use sample1.dta
. weaktsiv ry1 h* p* b* (ry2=z)
Two-sample Instrumental variables (TS2SLS) regression
First-stage F Results                                  Number of obs =  116901
---------------------------                            Wald chi2( 17) = 5611.78
F(  1, 10364) =   15.03                                Prob > chi2   =  0.0000
Prob > F      =  0.0001                                R-squared     =  0.1489
                                                       Adj R-squared =  0.1488
                                                       Root MSE      =  0.2205

------------------------------------------------------------------------------
         ry1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         ry2 |   .3716513   .1123615     3.31   0.001     .1514244    .5918781
       hdage |   .0192549   .0015888    12.12   0.000      .016141    .0223689
      hdage2 |  -.0198632   .0020119    -9.87   0.000    -.0238065   -.0159198
      hdmarr |   .0198959   .0062194     3.20   0.001      .007706    .0320859
    hdfemale |  -.0976462   .0107003    -9.13   0.000    -.1186186   -.0766738
     hdblack |  -.1211384   .0133036    -9.11   0.000    -.1472133   -.0950634
     hdother |  -.0163956     .00506    -3.24   0.001    -.0263131   -.0064781
      hdhisp |   .0005148   .0040625     0.13   0.899    -.0074478    .0084773
    hded0911 |   .0336821   .0052502     6.42   0.000     .0233919    .0439723
    hded1212 |   .0768111   .0054963    13.97   0.000     .0660384    .0875838
    hded1315 |   .1345398   .0068719    19.58   0.000     .1210711    .1480085
     hded16p |     .20465   .0078281    26.14   0.000     .1893071    .2199928
    pctlihtc |  -.7054565   .1697396    -4.16   0.000    -1.038143   -.3727696
      pctprj |  -.8651579   .1218703    -7.10   0.000    -1.104022   -.6262939
    pctrehab |  -1.026055    .160297    -6.40   0.000    -1.340235   -.7118757
      pctvch |  -2.920268   .5245491    -5.57   0.000    -3.948376    -1.89216
        boys |   -.004395   .0019092    -2.30   0.021     -.008137   -.0006529
       _cons |   .1192835   .0300903     3.96   0.000      .060307      .17826
------------------------------------------------------------------------------
Instrumented:  ry2
Instruments:   z
Confidence set and p-value for ry2 are based on normal approximation
------------------------------------------------------------------------------

------------------------------------------------------------------------------
                Weak IV Robust 95% confidence set and p-value
                         for H0: _b[ry2] = 0
------------------------------------------------------------------------------
 Test                          95% Confidence Set                      p-value
------------------------------------------------------------------------------
 Benchmark TSCLR              [ .2137159,  .7837229]                    0.0000
------------------------------------------------------------------------------
Note: In the just identifed case, TSCLR = TSAR = TSK.
```

Next, we illustrate the **weaktsiv** command in the over identified case using the dataset of Olivetti and Paserman (2015) and its specification in column (1) - row (5) in Table 3. The first half of the output again reports the classic two-sample 2SLS estimation results with Inoue and Solon (2010) standard errors. The second half of the output now reports results from benchmark TSAR, TSK, and TSCLR tests and confidence regions. The results are also reported in column (1) - row (5) in Table 2 of Choi et al. (2018).

```
. use sample2.dta
. weaktsiv ry1 (ry2=z*), level(90)
```

```
note: z726 omitted because of collinearity
Two-sample Instrumental variables (TS2SLS) regression
First-stage F Results                                  Number of obs =    16650
-----------------------                                Wald chi2(  1) =  153.60
F(726, 18771) =    1.98                                Prob > chi2   =  0.0000
Prob > F      =  0.0000                                R-squared     =  0.0098
                                                       Adj R-squared =  0.0097
                                                       Root MSE      =  0.4317

------------------------------------------------------------------------------
         ry1 |      Coef.   Std. Err.      t    P>|t|     [90% Conf. Interval]
-------------+----------------------------------------------------------------
         ry2 |    .354363   .0285928    12.39   0.000     .3073294    .4013965
       _cons |    1.92788   .0830654    23.21   0.000     1.791242    2.064518
------------------------------------------------------------------------------
Instrumented:  ry2
Instruments:   z1 z2 z3 z4 z5 z6 z7 z8 z9 z10 z11 z12 z13 z14 z15 z16 z17 z18
               z19 z20 z21 z22 z23 z24 z25 z26 z27 z28 z29 z30 z31 z32 z33 z34
               z35 z36 z37 z38 z39 z40 z41 z42 z43 z44 z45 z46 z47 z48 z49 z50
               z51 z52 z53 z54 z55 z56 z57 z58 z59 z60 z61 z62 z63 z64 z65 z66
               z67 z68 z69 z70 z71 z72 z73 z74 z75 z76 z77 z78 z79 z80 z81 z82
               z83 z84 z85 z86 z87 z88 z89 z90 z91 z92 z93 z94 z95 z96 z97 z98
               z99 z100 z101 z102 z103 z104 z105 z106 z107 z108 z109 z110 z111
               z112 z113 z114 z115 z116 z117 z118 z119 z120 z121 z122 z123 z124
               z125 z126 z127 z128 z129 z130 z131 z132 z133 z134 z135 z136 z137
               z138 z139 z140 z141 z142 z143 z144 z145 z146 z147 z148 z149 z150
               z151 z152 z153 z154 z155 z156 z157 z158 z159 z160 z161 z162 z163
               z164 z165 z166 z167 z168 z169 z170 z171 z172 z173 z174 z175 z176
               z177 z178 z179 z180 z181 z182 z183 z184 z185 z186 z187 z188 z189
               z190 z191 z192 z193 z194 z195 z196 z197 z198 z199 z200 z201 z202
               z203 z204 z205 z206 z207 z208 z209 z210 z211 z212 z213 z214 z215
               z216 z217 z218 z219 z220 z221 z222 z223 z224 z225 z226 z227 z228
               z229 z230 z231 z232 z233 z234 z235 z236 z237 z238 z239 z240 z241
               z242 z243 z244 z245 z246 z247 z248 z249 z250 z251 z252 z253 z254
               z255 z256 z257 z258 z259 z260 z261 z262 z263 z264 z265 z266 z267
               z268 z269 z270 z271 z272 z273 z274 z275 z276 z277 z278 z279 z280
               z281 z282 z283 z284 z285 z286 z287 z288 z289 z290 z291 z292 z293
               z294 z295 z296 z297 z298 z299 z300 z301 z302 z303 z304 z305 z306
               z307 z308 z309 z310 z311 z312 z313 z314 z315 z316 z317 z318 z319
               z320 z321 z322 z323 z324 z325 z326 z327 z328 z329 z330 z331 z332
               z333 z334 z335 z336 z337 z338 z339 z340 z341 z342 z343 z344 z345
               z346 z347 z348 z349 z350 z351 z352 z353 z354 z355 z356 z357 z358
               z359 z360 z361 z362 z363 z364 z365 z366 z367 z368 z369 z370 z371
               z372 z373 z374 z375 z376 z377 z378 z379 z380 z381 z382 z383 z384
               z385 z386 z387 z388 z389 z390 z391 z392 z393 z394 z395 z396 z397
               z398 z399 z400 z401 z402 z403 z404 z405 z406 z407 z408 z409 z410
               z411 z412 z413 z414 z415 z416 z417 z418 z419 z420 z421 z422 z423
               z424 z425 z426 z427 z428 z429 z430 z431 z432 z433 z434 z435 z436
               z437 z438 z439 z440 z441 z442 z443 z444 z445 z446 z447 z448 z449
               z450 z451 z452 z453 z454 z455 z456 z457 z458 z459 z460 z461 z462
               z463 z464 z465 z466 z467 z468 z469 z470 z471 z472 z473 z474 z475
               z476 z477 z478 z479 z480 z481 z482 z483 z484 z485 z486 z487 z488
               z489 z490 z491 z492 z493 z494 z495 z496 z497 z498 z499 z500 z501
               z502 z503 z504 z505 z506 z507 z508 z509 z510 z511 z512 z513 z514
               z515 z516 z517 z518 z519 z520 z521 z522 z523 z524 z525 z526 z527
               z528 z529 z530 z531 z532 z533 z534 z535 z536 z537 z538 z539 z540
               z541 z542 z543 z544 z545 z546 z547 z548 z549 z550 z551 z552 z553
               z554 z555 z556 z557 z558 z559 z560 z561 z562 z563 z564 z565 z566
               z567 z568 z569 z570 z571 z572 z573 z574 z575 z576 z577 z578 z579
               z580 z581 z582 z583 z584 z585 z586 z587 z588 z589 z590 z591 z592
               z593 z594 z595 z596 z597 z598 z599 z600 z601 z602 z603 z604 z605
               z606 z607 z608 z609 z610 z611 z612 z613 z614 z615 z616 z617 z618
               z619 z620 z621 z622 z623 z624 z625 z626 z627 z628 z629 z630 z631
               z632 z633 z634 z635 z636 z637 z638 z639 z640 z641 z642 z643 z644
               z645 z646 z647 z648 z649 z650 z651 z652 z653 z654 z655 z656 z657
               z658 z659 z660 z661 z662 z663 z664 z665 z666 z667 z668 z669 z670
               z671 z672 z673 z674 z675 z676 z677 z678 z679 z680 z681 z682 z683
               z684 z685 z686 z687 z688 z689 z690 z691 z692 z693 z694 z695 z696
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              z697 z698 z699 z700 z701 z702 z703 z704 z705 z706 z707 z708 z709
              z710 z711 z712 z713 z714 z715 z716 z717 z718 z719 z720 z721 z722
              z723 z724 z725 z726
Confidence set and p-value for ry2 are based on normal approximation
```

```
               Weak IV Robust 90% confidence sets and p-values
                          for H0: _b[ry2] = 0
```

Test	90% Confidence Set	p-value
Benchmark TSCLR	[.5706445, .7306929]	0.0000
Benchmark TSAR	empty	0.0000
Benchmark TSK	[-2.803132, -2.362719] U [.5633191, .7388234]	0.0000

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