Two-Sided Heterogeneity, Endogenous Sharing, and International Matching Markets

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Abstract

This paper develops a multi-country, multi-sector, and multi-factor model of two-sided matching between heterogeneous workers and entrepreneurs in which agents in different countries can form cross-country teams. Sorting, matching, and sharing problems are all considered in a unified framework. Equilibrium is characterized by endogenous sharing rules, which break away from competitive marginal productivity theories of factor returns. I illustrate that a reduction in the cost of sector-specific matching can increase welfare for all agents without conflicts of interest, and that a bilateral economic integration agreement can affect the welfare of agents in an unrelated third country. Furthermore, I demonstrate that rising income inequality can be accompanied by strengthening negative assortative matching.

Keywords: Two-Sided Matching; Sorting; Matching; Offshoring; Economic Integration Agreement
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1 Introduction

Interest in two-sided matching models is burgeoning in the international trade literature (Eaton, Jinkins, Tybout and Xu, 2016; Krolikowski and McCallum, 2018; Bernard, Moxnes and Ulltveit-Moe, 2018). In order for heterogeneous firms to find new matching partners to sell to or buy goods from in foreign markets, they must incur a search cost or a relationship-specific fixed cost to match with each partner. Most current studies of two-sided matching models in the international trade literature deal with this exporter-importer matching problem (in the international goods market). Likewise, as foreign direct investment around the world has increased tremendously with rapid advancements in transportation and communication technologies, studies have investigated why entrepreneurs hire foreign workers to produce, who matches whom, and the distributional consequences for both heterogeneous entrepreneurs and heterogeneous workers (in the international factor market).

However, there are fewer studies of two-sided cross-country matching between heterogeneous entrepreneurs and heterogeneous workers (Kremer and Maskin, 2006; Antràs, Garicano and Rossi-Hansberg, 2006; Choi, 2019). Moreover, the existing studies that analyze the international two-sided matching problem in factor markets are based on a two-country framework and investigate a move from autarky to complete globalization because adding multiple dimensions (> 2) to the model is complicated. It would be useful to know what the distributional impacts are of country A and country B signing a bilateral investment treaty (i.e., an easing of cross-country matching between country A and country B) on agents in country C. Or what the distributional impacts are of incomplete integration, such as a reduction in the costs of cross-country matching instead of complete globalization.

In this paper, I fill this gap in the literature by developing a multi-country, multi-sector, and multi-factor model of two-sided matching between heterogeneous workers and entrepreneurs in which agents in different countries can form cross-country teams. To overcome the methodological challenge, I borrow this paper's framework from both the two-sided matching literature and the international trade literature. Specifically, I extend Galichon, Kominers and Weber (2019)'s two-sided matching model to allow for multiple countries and multiple sectors, as in Costinot (2009), by considering labor matching markets and goods markets simultaneously. This makes it possible to answer questions in international economics using the tools and techniques developed in the two-sided matching literature. From an international trade literature perspective, I extend the multi-country, multi-sector, and multi-factor neoclassical trade model of Costinot (2009) to allow for cross-country matching and a complementarity effect between factors with a sharing problem.

More specifically, the methodological challenge that I address in this paper is to build a model that analyzes the impact of falling costs of offshoring (or cross-country matching) in a multi-country framework while assuming the two-sided heterogeneous structure of factors of production.[1] Standard international trade models assume homogeneous labor in each country (Eaton and Kortum, 2002; Melitz, 2003). Admittedly, there are more recent theoretical advances that incorporate heterogeneity of workers into these two workhorse trade models. However, to the best of my knowledge, Kremer and Maskin (2006), Antràs, Garicano and Rossi-Hansberg (2006), and Choi (2019) are three exceptions that analyze the impact of the cross-country matching problem in a two-country framework.[2] A cross-country matching problem is modeled as a move from (complete) autarky to (complete) globalization in a two-country framework in Antràs, Garicano and Rossi-Hansberg (2006) and in Choi (2019). Adding one more country to these models is technically challenging and also does not suggest more meaningful economic implications. This paper expands this strand of research to allow for multiple countries.

A natural starting point to think about modeling a multi-country framework in the international trade literature is to use a Fréchet distribution (the Type II extreme value distribution) as in Eaton and Kortum (2002). However, the role of the Fréchet distribution in their framework is to employ a probabilistic representation of technologies that can relate trade flows to underlying parameters in a multi-country framework. Hence, Eaton and Kortum (2002)'s framework may not be adequate to tackle the research question. I turn my attention to the two-sided matching literature, which already features two-sided heterogeneity, and try to incorporate a

[1]Suppose that there are three countries in the world. In each country, there are heterogeneous workers and entrepreneurs. What would happen if the costs of forming cross-country teams (such as a team of country-1 entrepreneurs paired up with country-2 workers) decreased due to improvements in information technology or a bilateral investment treaty between country-1 and country-2? Does inequality within each country increase or decrease? To answer these questions, the model should feature multi-country and two-sided heterogeneous multi-factor components.

[2]Note that Antràs, Garicano and Rossi-Hansberg (2006) extend a knowledge-based hierarchy model of Garicano (2000) to allow for cross-country matching; Choi (2019) extends the matching framework in Grossman, Helpman and Kircher (2017) and in Eeckhout and Kircher (2018) to allow for cross-country matching.

multi-country framework into it. In this vein, I begin by basing my model on Choo and Siow (2006)'s transferable utility model of the marriage market, where they adopt a Logit distribution (the Type I extreme value distribution) following McFadden (1974). Choo and Siow (2006) already features two-sided heterogeneity. In addition, incorporating a multi-country (along with a multi-sector) feature into Choo and Siow (2006) is tractable due to the Logit distribution. One disadvantage of adopting Choo and Siow (2006) is that the transfer of utility is linear, which turns out to be a bit restrictive in my application. Galichon, Kominers and Weber (2019) propose a general framework for models of one-to-one matching with an imperfectly transferable utility that includes Choo and Siow (2006) as a special case. I therefore borrow elements from Galichon, Kominers and Weber (2019) to build a multi-country, multi-sector, and multi-factor model of two-sided matching between heterogeneous workers and entrepreneurs.

There are several notable features in the model. First, labor matching markets and goods markets are considered simultaneously in the framework. An equilibrium is defined as a pair of labor matching market-clearing wages and goods market-clearing prices. Typical two-sided matching studies consider a worker-entrepreneur match in one labor market, separate from other markets. I consider a match between workers and entrepreneurs together with markets for their produced goods where the workers and the entrepreneurs are also the consumers. This allows me to analyze how goods market-specific shocks influence labor matching markets, and vice versa.

Second, I consider voluntary unemployment in which some agents choose not to work in an equilibrium. Typical general equilibrium models consider a full-employment condition in labor markets. In this paper, however, each agent has a heterogeneous preference to work with different types of partners in different sectors or to remain unmatched. I consider a type I extreme value distribution for the preference heterogeneity of each agent.[3] Some agents have a strong preference for remaining unmatched. An advantage of modeling unemployment in an equilibrium is that it is possible to analyze how globalization affects unemployment levels.

Third, a sharing rule in each one-to-one match is endogenously determined, which breaks from competitive marginal productivity theories of factor returns. More specifically, in a matching market between type-θ workers and type-ρ en-

[3]The support of the distribution is unbounded.

trepreneurs in sector s, two sufficient statistics determine the sharing rule: the quantity of unmatched type-θ workers and the quantity of type-ρ entrepreneurs. The sharing rule is similar to that of Rubinstein and Wolinsky (1985) who find that, in a random matching with a sequential bargaining process, bargaining power is determined by the relative size of buyers and sellers as players become infinitely patient. However, unlike in Rubinstein and Wolinsky (1985), the relative quantity of unmatched workers and unmatched entrepreneurs is endogenously determined. Using this novel sharing rule, I study how globalization affects the bargaining power of each agent, which ultimately feeds into factor returns.

Fourth, I derive closed-form expressions of a wage function and a matching function that depend on the quantity of unmatched workers of each type, the quantity of unmatched entrepreneurs of each type, and the price of a good given an exogenous production function. This tractable representation allows me to conduct several comparative static exercises that investigate how changes in production technology (such as a reduction in the cost of sector-specific matching, a bilateral economic integration agreement, and a skill-biased technical change) affect matching patterns and sharing rules.

Fifth, I can measure the welfare of each agent by calculating the expected indirect payoff before he observes his realizations of a vector of preference heterogeneity. Suppose that a worker of type θ (or an entrepreneur of type ρ) is uncertain about his or her matching partner, including the option of remaining unmatched. I derive ex ante expected indirect payoffs for type-θ workers and type-ρ entrepreneurs, respectively, and demonstrate that the formula is expressed as a logsum, which is identical to the welfare formula of Small and Rosen (1981).[4] Type-level welfare metrics are then aggregated up to the country-level and again aggregated up to the world level. In addition, the social welfare formula, calculated from the summation of ex-ante expected indirect payoffs, is "inequality-adjusted," as in Jones and Klenow (2016) in the macroeconomics literature and Galle, Rodriguez-Clare and Yi (2017) and Antras, De Gortari and Itskhoki (2017) in the international trade literature. With the social welfare formula in the model, I conduct a systematic welfare analysis of the impact of globalization: Who benefits and who loses from globalization?

Two simple comparative statics shed light on the consequences of globalization. First, I explore the impacts of reductions in sector-specific matching costs. I iden-

[4]They study measurement of welfare changes in a discrete choice model.

tify a productivity effect, a relative price effect, and a labor supply effect that are similar to mechanisms in the task-based offshoring model of Grossman and Rossi-Hansberg (2008). Suppose that matching costs in sector s decrease. In sector s, the cost reduction of matching leads to an increase in team productivity and hence induces more agents to match and produce good s. In other sectors, the equilibrium terms of trade changes and relative prices increase, which increases the total surplus in each match. The relative price effect negatively affects sector s, but I show that the net effect in sector s is always positive and is greater than the positive relative price effect in other sectors. Therefore, for all types of agents, the quantity of those that are unmatched is reduced – i.e., the labor supply effect – which raises the welfare of all agents.

Next, I study the implications of bilateral economic integration, such as country 1 and country 2 signing a bilateral investment treaty, on agents in country 3, which reduces the costs of matching between country-1 agents and country-2 agents. I find that, as expected, the welfare of country-3 agents diminishes when such a bilateral investment treaty is in effect. The reason is that because there are more profitable matching options available for country-1 and country-2 agents, the quantity of unmatched agents is reduced. The bargaining power of country-3 agents declines in cross-country matching markets (such as a matching market between country-1 and country-2 agents) because there are fewer available country-1 and country-2 agents.

The reduced bargaining power of country-3 agents affects matching patterns in country-3 matching markets non-monotonically. Interestingly, the amount of within-country matching rises in country 3. The reason is that some country-3 agents who used to match with country-1 (or country-2) agents now revert to country 3 from cross-country matching markets (the phenomenon of reshoring) to form production teams between country-3 workers and country-3 entrepreneurs. The results are reminiscent of the trade creation and trade diversion phenomena identified by Viner (1950).

In addition to the two examples in the international trade literature, I investigate a relationship between assortative matching and income inequality because the model features tractable expressions of wage and matching functions. In a 2×2 stochastic Becker (1973, 1974)-type setting, I find that rising income inequality is not necessarily in tandem with increasing (stochastic) positive assortative matching.

2 Related Literature

Cross-Country Matching. This paper is closely related to the work of Kremer and Maskin (2006), Antràs, Garicano and Rossi-Hansberg (2006), and Choi (2019), who modeled the globalization of production processes as a cross-country matching between agents and studied the distributional impacts of globalization in each country. Whereas the previous studies consider a two-country framework, this paper extends the previous cross-country matching framework to an arbitrary number of countries. A multi-country framework enables me to analyze the impacts of lower costs of matching between two countries on unrelated third-country parties: a network effect or a general equilibrium effect. Second, whereas the earlier studies deal with a move from autarky to complete globalization, here I can analyze a finite drop in cross-country matching costs.

Multi-Country Framework. Admittedly, there are some notable multi-country theoretical models in the international trade literature such as Eaton and Kortum (2002). They use a Fréchet distribution (the Type II extreme value distribution) to extend a two-country Ricardian model with a continuum of goods to a multi-country and multi-good framework. Rodríguez-Clare (2010) allows for fragmentation and offshoring in Eaton and Kortum (2002)'s framework. However, both studies assume that labor is homogeneous in each country and hence the models cannot address the impact of offshoring and/or cross-country matching on wage distribution within a country. To the best of my knowledge, Costinot (2009) considers a multi-factor generalization of the Ricardian model to predict the patterns of international specialization using tools from the mathematics of complementarity (i.e., log-supermodularity). However, there are some differences between Costinot (2009)'s framework and my model. First, Costinot (2009) does not consider cross-country matching, which is the core component in my framework. Second, Costinot (2009) rules out imperfect substitutability between factors within each sector. In his model, a certain type of factor is allocated to a certain sector (the sorting problem). But factors of production are perfect substitutes within each country and sector (no matching problem). In my framework, factors are allocated to sectors, and simultaneously they should find a partner (either a worker or an entrepreneur) to produce a good. Hence, both a sorting problem and a matching problem are considered in the model.

Two-Sided Matching. Another related research area is the two-sided marriage matching market. Choo and Siow (2006) propose a stochastic version of Becker (1973, 1974)'s classic static transferable utility model of the marriage markets by incorporating random identically distributed McFadden (1974)-type noise in the preferences of each of the participants. More recently, Galichon, Kominers and Weber (2019) provide a general framework of an imperfectly transferable utility model with preference heterogeneity in tastes. This paper extends Galichon, Kominers and Weber (2019)'s framework to allow for multiple countries and multiple sectors. Hence, labor matching markets and goods markets are considered simultaneously in a unified framework, whereas typical matching studies consider the labor market separately from other markets.

My model's matching function is related to Mortensen and Pissarides (1994)-type constant returns to scale reduced-form matching function $m(u, v)$. Unlike Mortensen and Pissarides (1994), my model endogenously derives a matching function with constant returns to scale by solving the discrete choice problems of both sides of the market.

Bargaining Power. My model is to some extent related to the work of Rubinstein and Wolinsky (1985), who study a matching and sequential bargaining problem. Rubinstein and Wolinsky (1985) argue that, when agents are infinitely patient, the bargaining power between a buyer and a seller can be represented as the ratio between the number of buyers and the number of sellers. I consider a frictionless searching process of sorting, matching, and sharing and find that the sharing rule (or the bargaining power) in each one-to-one match is similarly expressed as the ratio between the quantity of unmatched entrepreneurs and the quantity of unmatched workers.

In addition, Gale (1986a,b, 1987) argues that any perfect equilibrium of the bargaining game implements a Walras allocation of the exchange economy. Similar to his insight, equilibrium in my model is a Walrasian equilibrium with a wage vector w and a price vector p. Alternatively, the model can be interpreted as a sorting, matching, and sharing framework in which a large number of agents form a production team in a sector and then bargain over a set of feasible utilities in one-to-one matching. In a frictionless setting with a given price vector p, I find a pairwise stable matching μ that implements a Walras allocation of the economy.

3 The Model

The model builds on the framework of Galichon, Kominers and Weber (2019), who propose a static imperfectly transferable utility model of two-sided one-to-one matching. I extend their matching framework to allow for multiple countries and multiple sectors by considering labor matching markets and goods markets simultaneously. I uncover a closed-form expression of endogenous sharing rules between two agents in a pairwise stable equilibrium. From an international trade theory perspective, I extend the multi-country, multi-sector, and multi-factor neoclassical trade model of Costinot (2009) to allow for cross-country matching and a complementarity effect between factors with a sharing problem.

3.1 Environment

3.1.1 Agents

There are G countries indexed by $g, h \in \mathcal{G} := \{1, 2, ..., G\}$ and S sectors (or goods) indexed by $s \in \mathcal{S} := \{1, 2, ..., S\}$ in the world. There are two sets of agents: workers and entrepreneurs. Let $\mathcal{X}$ and $\mathcal{Y}$ be the finite sets of characteristics of workers and entrepreneurs, where worker characteristics are indexed by $x \in \mathcal{X}$ and entrepreneur characteristics are indexed by $y \in \mathcal{Y}$. There are X number of characteristics of workers and Y number of characteristics of entrepreneurs.[5] I define the sets of *types* of workers and entrepreneurs as Cartesian products $\mathcal{X} \times \mathcal{G}$ and $\mathcal{Y} \times \mathcal{G}$, respectively. Assume that agents are clustered in groups of similar types but heterogeneous preferences. Let $xg_i \in \mathcal{X} \times \mathcal{G}$ be the type of individual worker i whose characteristic is x and resides in country g.[6] For each $xg \in \mathcal{X} \times \mathcal{G}$, we let n_{xg} be the quantity of inelastic workers of type xg. Likewise, let $yh_j \in \mathcal{Y} \times \mathcal{G}$ be the type of individual entrepreneur j whose characteristic is y and resides in country h. For each $yh \in \mathcal{Y} \times \mathcal{G}$, we let m_{yh} be the quantity of inelastic entrepreneurs of type yh. I assume that there is a sufficiently large number of agents of each type, denoted as "Large Matching Markets" (See Choo and Siow, 2006; Kojima, Pathak and Roth, 2013; Galichon and Salanié, 2015; Menzel, 2015; Azevedo and Leshno, 2016; Lee, 2016) and that agents' types are publicly observable.

[5] $|\mathcal{X}| = X$ and $|\mathcal{Y}| = Y$.

[6] xg is defined as an ordered pair (x, g) where $x \in \mathcal{X}$ and $g \in \mathcal{G}$.

Workers and entrepreneurs can freely choose to work in any sector s (sorting problem). Once they choose a certain sector s, then they form pairs and produce good s (matching problem).[7] Note that workers and entrepreneurs cannot move across countries–i.e., country is an attribute of the worker and the entrepreneur, and immigration is not allowed in the model. A novel feature of the model is that it allows cross-country matching between workers and entrepreneurs. For instance, a worker who resides in country g can work with an entrepreneur who resides in country h (through offshoring or FDI).[8]

3.1.2 Surplus Function

If a worker $xg \in \mathcal{X} \times \mathcal{G}$ and an entrepreneur $yh \in \mathcal{Y} \times \mathcal{G}$ are matched in sector $s \in \mathcal{S}$, they can jointly produce $q_{xg,yh,s}$ units of good s. A vector $q = (q_{xg,yh,s})_{xg \in \mathcal{X} \times \mathcal{G}, yh \in \mathcal{Y} \times \mathcal{G}, s \in \mathcal{S}}$ defines a production function. Note that the output $q_{xg,yh,s}$ does not depend on individual heterogeneity – i.e., i or j.[9] I assume a perfectly competitive goods market. Also, I assume that goods can freely move across borders, which ensures that the world price of good s is given by $p_s > 0$. A joint surplus (or revenue) of a team is given by $p_s q_{xg,yh,s}$. After they create a joint surplus, they bargain over how to divide the joint surplus between them.

I denote a transfer from an entrepreneur $yh \in \mathcal{Y} \times \mathcal{G}$ to a worker $xg \in \mathcal{X} \times \mathcal{G}$ in sector $s \in \mathcal{S}$ as $w_{xg,yh,s} \in \mathbb{R}$. If both agents agree, then the joint surplus is frictionlessly divided between the worker and the entrepreneur. The share for the worker (denoted as wage) is $w_{xg,yh,s} \in \mathbb{R}$ and the share for the entrepreneur (denoted as profit)

[7] Following the terminologies in Grossman (2013), I use "sorting" to refer to the allocation of resources between sectors of the economy and "matching" to refer to the allocation of resources within sectors. The sorting (without matching) problem has been studied in its most general form in Costinot (2009); while most marriage matching models focus on the matching (without sorting) problem. Grossman, Helpman and Kircher (2017) consider both matching and sorting problems in two goods and two factors of production in which both workers and managers are heterogeneous in their abilities. In this paper, I consider both matching and sorting problems in a multi-country, multi-sector, and multi-factor framework that allows for cross-country matching.

[8] An example of offshoring is when US-based multinational companies to produce their goods in China. In this case, US managers are paired up with Chinese workers.

[9] A simple way to incorporate the costs of cross-country matching (such as costs of offshoring or FDI) into the production function is in terms of iceberg trade costs: $q_{xg,yh,s} = \frac{\psi_{xg,yh,s}}{\tau_{g,h,s}}$ where $\tau_{g,h,s}$ denotes the iceberg-type costs of matching when a worker in country g and an entrepreneur in country h produce good s. Alternatively, one can model the cost of cross-country matching taking the form of fixed costs: $q_{xg,yh,s} = \psi_{xg,yh,s} - f_{g,h,s}$ where $f_{g,h,s}$ indicates the fixed costs of matching when a worker in country g and an entrepreneur in country h produce good s.

is $\pi_{xg,yh,s} := p_s q_{xg,yh,s} - w_{xg,yh,s} \in \mathbb{R}$. I assume a complete contract environment between a worker and an entrepreneur. If either side rejects the agreement, then the worker and the entrepreneur break up and search for new partners independently. It is assumed that all agents are infinitely patient. This implies that a new search is costless regardless of the number of searches. Agents can search and bargain over a joint surplus as long as they like.

3.1.3 Utilities

Suppose that a worker xg_i matches with an entrepreneur yh_j in sector s. The wage is given by $w_{xg,yh,s}$ and the profit is given by $\pi_{xg,yh,s}$. Let us further assume that the utilities of worker and entrepreneur are respectively given by:

$$u_{xg_i,yh_j,s} = \theta \ln \left[\sum_s q_s^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} + \varepsilon_{xg_i,yh_j,s}$$
$$v_{xg_i,yh_j,s} = \theta \ln \left[\sum_s x_s^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} + \eta_{xg_i,yh_j,s}$$

where $\theta > 0$ denotes a relative weight on the consumption part of the utility, $\sigma > 1$ denotes the elasticity of substitution between goods, q_s is the consumption of good s by the worker xg_i who receives wage $w_{xg,yh,s}$, x_s is the consumption of good s by the entrepreneur yh_j who receives profit $\pi_{xg,yh,s}$, and $\varepsilon_{xg_i,yh_j,s}$ represents the worker xg_i's heterogeneous preference to work with entrepreneur yh_j in sector s, and $\eta_{xg_i,yh_j,s}$ denotes the entrepreneur yh_j's heterogeneous preference to work with worker xg_i in sector s.

In the goods market, given a price vector $p = (p_s)_{s\in\mathcal{S}}$, each worker and entrepreneur makes a consumption choice under a budget constraint. A type-xg worker, who matches with type-yh entrepreneur, produces good s, and receives wage $w_{xg,yh,s}$ solves:

$$\max_{(q_s)} \theta \ln \left[\sum_s q_s^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad \text{subject to} \quad \sum_s p_s q_s \leq w_{xg,yh,s}.$$

Consumption for good s is given by,

$$q_s = \frac{p_s^{-\sigma}}{P^{1-\sigma}} w_{xg,yh,s}$$

where P is the price index with $P^{1-\sigma} := \sum_s p_s^{1-\sigma}$. Plugging the consumption of good s into the utility of worker, I can represent the utility of worker xg_i who matches with entrepreneur yh_j in sector s as follows (according to the duality theorem):

$$u_{xg_i,yh_j,s} = \theta \ln \frac{w_{xg,yh,s}}{P} + \varepsilon_{xg_i,yh_j,s}.$$

Using the same step, entrepreneur yh_j who matches with worker xg_i in sector s has the following utility:

$$v_{xg_i,yh_j,s} = \theta \ln \frac{\pi_{xg,yh,s}}{P} + \eta_{xg_i,yh_j,s}.$$

The log utility function plays a key role in the model from two perspectives. First, conceptually, the transfers between workers and entrepreneurs are imperfect, and hence the model is different from the perfectly transferable utility model of Choo and Siow (2006), where the transfer of utility is linear. In my model, the consumption part of the utility corresponds to the concept of income (i.e., wage and profit). Therefore, a linear mapping from income to utility may not be appropriate. Hence, I assume that the marginal utility of income declines as income increases. This also implies that the utility cost of a concession to one party may not be equal to the benefit of another party (Galichon, Kominers and Weber, 2019). Second, technically, the log utility function has more useful properties compared to the linear utility function used by Choo and Siow (2006). In equilibrium, I will derive a closed-form solution of endogenous sharing rules that are represented by two functions: wage function and profit function. Unlike the linear utility function where a wage and a profit can have negative values (or have greater values than the total surplus), the wage and the profit are bounded below by zero and above by the total surplus.

If worker xg_i and entrepreneur yh_j decide to remain unmatched, they get reservation utilities respectively as follows:

$$U_{xg_i,0} = \theta \ln \frac{w_{xg,0}}{P} + \varepsilon_{xg_i,0} \quad \text{and} \quad V_{0,yh_j} = \theta \ln \frac{\pi_{0,yh}}{P} + \eta_{0,yh_j}$$

where $w_{xg,0}$ and $\pi_{0,yh}$ are unemployment benefits (i.e., outside options) for type-xg worker and type-yh entrepreneur, respectively. Let us assume that the unemployment benefit level is not type-dependent.[10] Without loss of generality, I further assume that $w_{xg,0} = \pi_{0,yh} = P$ for all $xg \in \mathcal{X} \times \mathcal{G}$ and $yh \in \mathcal{Y} \times \mathcal{G}$ such that:

$$U_{xg_i,0} = \varepsilon_{xg_i,0} \quad \text{and} \quad V_{0,yh_j} = \eta_{0,yh_j}.$$

3.1.4 Preference Heterogeneity

Assume that worker xg_i's preference heterogeneity does not depend on the entrepreneur's identity j. This implies that, if worker xg_i decides to match with yh entrepreneur in sector s, then his preference to work with yh_j or yh_k is indifferent, i.e., $\varepsilon_{xg_i,yh_j,s} = \varepsilon_{xg_i,yh_k,s}$. Hence, the dimension of preference heterogeneity (the choice set) is reduced from an individual-sector level to a type-sector level. I can define worker xg_i's preference heterogeneity as a $(YGS + 1) \times 1$ vector:

$$\varepsilon_{xg_i} = (\varepsilon_{xg_i,0}, \varepsilon_{xg_i,yh,s})_{yh \in \mathcal{Y} \times \mathcal{G}, s \in \mathcal{S}}.$$

Likewise, I can define entrepreneur yh_j's preference heterogeneity as a $(XGS+1)\times 1$ vector:

$$\eta_{yh_j} = (\eta_{0,yh_j}, \eta_{xg,yh_j,s})_{xg \in \mathcal{X} \times \mathcal{G}, s \in \mathcal{S}}.$$

I assume that each component of a preference heterogeneity vector is an independently and identically distributed random variable with a type I extreme value distribution as follows:

$$F(\varepsilon) = \exp\left(-\exp\left(-(\varepsilon + \gamma)\right)\right),$$

where the mean is given by $E(\varepsilon) = 0$ and $\gamma \approx 0.577$, Euler's constant, and the variance is given by $V(\varepsilon) = \dfrac{\pi^2}{6}$ where $\pi \approx 3.14$.[11] The stochastic part ensures that workers of type xg can match with different types of entrepreneurs in an equilibrium. Furthermore, some agents end up remaining unmatched because the support of the distribution is $(-\infty, \infty)$, implying that all combinations of matches can be observed in an equilibrium.

[10]One fruitful extension would be to model unemployment benefits to depend on types and/or countries. In such a case, it would be possible to analyze the impact of unemployment benefit policy on matching and sharing patterns. For instance, if the US government changes Unemployment Insurance (UI) Program, the model can predict how such a policy change affects the welfare of agents in the US and other countries in a multi-country framework.

[11]Note that we change the location parameter of a standard Gumbel distribution to set the expected value as zero.

The independence of irrelevant alternatives (IIA) assumption of the conditional Logit model appears to be a strong restriction on the model structure. One can embed correlations across choices, $0 < \rho \leq 1$, into both worker's preference heterogeneity and entrepreneur's heterogeneity, with $\rho = 1$ implying independence. In this case, the equilibrium conditions stand still, with θ replaced with $\frac{\theta}{\rho}$. More realistically, the preference heterogeneity can follow a two-level nested Logit model where choices are correlated within the nest (either sector or country); choices are independent across the nests. However, in the two-level nested Logit case, conditional choice probabilities become complicated and I lose analytical tractability.[12] In addition, the focus of this paper is to analyze the impact of the falling cost of cross-country matching, which is a component of production structure, not of taste heterogeneity. Hereafter, therefore, I assume the independence irrelevant alternatives (IIA) of the conditional Logit framework.

3.1.5 Indirect Payoffs

Let u_{xg_i} and v_{yh_j} be the indirect payoff of worker xg_i and entrepreneur yh_j, respectively. Given a wage vector $w = (w_{xg,yh,s})_{xg\in\mathcal{X}\times\mathcal{G},yh\in\mathcal{Y}\times\mathcal{G},s\in\mathcal{S}}$ and a price vector $p = (p_s)_{s\in\mathcal{S}}$, the indirect payoffs are represented as follows:

$$u_{xg_i} = \max_{\substack{yh\in\mathcal{Y}\times\mathcal{G}\\ s\in\mathcal{S}}} \{\theta \ln \frac{w_{xg,yh,s}}{P} + \varepsilon_{xg_i,yh,s}, \varepsilon_{xg_i,0}\},$$

$$v_{yh_j} = \max_{\substack{xg\in\mathcal{X}\times\mathcal{G}\\ s\in\mathcal{S}}} \{\theta \ln \frac{\pi_{xg,yh,s}}{P} + \eta_{xg,yh_j,s}, \eta_{0,yh_j}\},$$

where $\pi_{xg,yh,s} := p_s q_{xg,yh,s} - w_{xg,yh,s}$. Each agent maximizes the indirect payoff by matching with a partner in a sector or by remaining unmatched. Note that worker xg_i has $YGS + 1$ number of strategies and entrepreneur yh_j has $XGS + 1$ number of strategies. The dimension of the strategy set reduces from an individual level to a type-sector level due to the assumptions that the production function $q_{xg,yh,s}$ does not depend on individual heterogeneity (i.e., i or j) and preference heterogeneity does not depend on the partner's identity.

[12] See Small (1987) and Eaton and Kortum (2002) for more details.

3.1.6 Feasible Bargaining Set

I establish a structure of the feasible bargains among production teams. Let $U_{xg,yh,s}$ and $V_{xg,yh,s}$ be the consumption parts of the utility for workers of type xg and entrepreneurs of type yh who would form a production team in sector s. They bargain over a set of feasible utilities $(U, V) \in \mathcal{F}_{xg,yh,s}$, where a feasible bargaining set $\mathcal{F}_{xg,yh,s}$ is defined as follows:[13]

$$\mathcal{F}_{xg,yh,s} := \left\{ (U, V) \in \mathbb{R}^2 \;\middle|\; (\exp(U))^{1/\theta} + (\exp(V))^{1/\theta} \leq \frac{p_s q_{xg,yh,s}}{P} \right\}. \tag{1}$$

A novel feature of the feasible bargaining set $\mathcal{F}_{xg,yh,s}$ is that prices of goods are included in the set; in other words, goods market conditions interact with a bargaining problem in labor matching markets. Define $\mathcal{U}_{xg,yh,s}(w_{xg,yh,s}; p) := \theta \ln \frac{w_{xg,yh,s}}{P}$ and $\mathcal{V}_{xg,yh,s}(w_{xg,yh,s}; p) := \theta \ln \frac{p_s q_{xg,yh,s} - w_{xg,yh,s}}{P}$ as utilities after transfer where $\mathcal{U}_{xg,yh,s}(w_{xg,yh,s}; p)$ is a continuous and nondecreasing function and $\mathcal{V}_{xg,yh,s}(w_{xg,yh,s}; p)$ is a continuous and nonincreasing function.

Figure 1 illustrates a feasible bargaining set $\mathcal{F}_{xg,yh,s}$ when $\frac{p_s q_{xg,yh,s}}{P} = 100$ and $\theta = 0.5$. Unlike a transferable utility model, the slope of the bargaining frontier is not a straight line. The source of non-linearity originates from the functional form of the utility function — i.e., the log utility function.

3.1.7 Matching

Let $\mu_{xg,yh,s}$ be the quantity of matches between workers of type xg and entrepreneurs of type yh in sector s. A matching is defined as a vector $\mu = (\mu_{xg,yh,s})_{xg \in \mathcal{X}\times\mathcal{G}, yh \in \mathcal{Y}\times\mathcal{G}, s \in \mathcal{S}}$ that satisfies

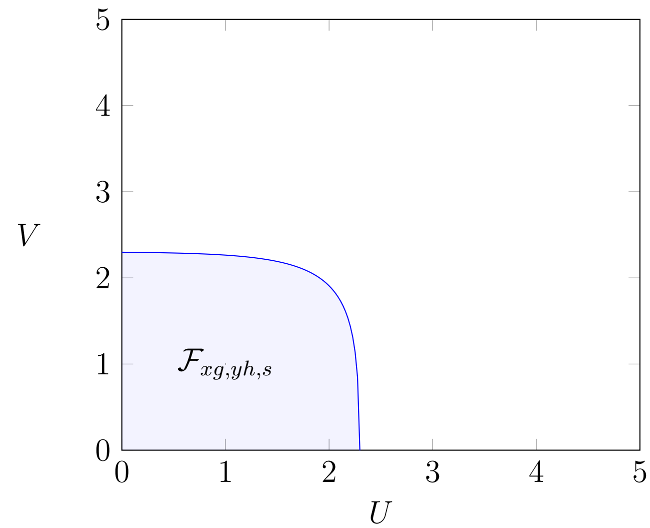
$$\mu_{xg,yh,s} \geq 0, \qquad \sum_{\substack{yh \in \mathcal{Y}\times\mathcal{G} \\ s \in \mathcal{S}}} \mu_{xg,yh,s} \leq n_{xg}, \quad \text{and} \quad \sum_{\substack{xg \in \mathcal{X}\times\mathcal{G} \\ s \in \mathcal{S}}} \mu_{xg,yh,s} \leq m_{yh}$$

for all $xg \in \mathcal{X} \times \mathcal{G}$, $yh \in \mathcal{Y} \times \mathcal{G}$, and $s \in \mathcal{S}$.

For any matching, I denote u_i and v_j as the indirect payoff to worker i and entrepreneur j, respectively. Also, let $U_{i,0}$ and $V_{0,j}$ be the reservation utilities for worker i and entrepreneur j, respectively.

[13] Note that the set $\mathcal{F}_{xg,yh,s}$ is a proper bargaining set as in Galichon, Kominers and Weber (2019): (a) $\mathcal{F}_{xg,yh,s}$ is closed and empty, $\mathcal{F}_{xg,yh,s}$ is lower comprehensive, and (c) $\mathcal{F}_{xg,yh,s}$ is bounded above.

Figure 1: Feasible Bargaining Set and Pareto Efficient Frontier



Notes: We assume that $\frac{p_s q_{xg,yh,s}}{P} = 100$ and $\theta = 0.5$.

Definition 1. *A matching μ is stable if there exist a pair (w, p) such that*

i) Individual Rationality : For all workers i and entrepreneurs j who are matched, $u_i \geq U_{i,0}$ and $v_j \geq V_{0,j}$,

ii) Pairwise Stability : There is no blocking coalition (i, j) of workers and entrepreneurs who would be able to reach a feasible pair of indirect payoffs dominating u_i and v_j.

Following Choo and Siow (2006)'s original insight, finding a stable matching is equivalent to solving discrete choice problems on both sides of the market. Hence, I follow the same steps as in Choo and Siow (2006) to characterize a stable matching.

3.2 Discrete Choice Problems

Worker xg_i will maximize his (or her) indirect payoff by choosing an entrepreneur in a sector among $YGS + 1$ number of available matching alternatives:

$$u_{xg_i} = \max_{\substack{yh \in \mathcal{Y} \times \mathcal{G} \\ s \in \mathcal{S}}} \{\theta \ln \frac{w_{xg,yh,s}}{P} + \varepsilon_{xg_i,yh,s}, \varepsilon_{xg_i,0}\}. \tag{2}$$

Let $\Pr\left[u_{xg_i} = \theta \ln \frac{w_{xg,yh,s}}{P} + \varepsilon_{xg_i,yh,s}\right]$ be the probability of choosing a type yh entrepreneur in sector s and $\Pr[u_{xg_i} = \varepsilon_{xg_i,0}]$ be the probability of remaining unmatched. Following McFadden (1974), I can derive conditional choice probabilities as follows (see Appendix 6.1 for a detailed derivation):

$$\Pr\left[u_{xg_i} = \theta \ln \frac{w_{xg,yh,s}}{P} + \varepsilon_{xg_i,yh,s}\right] = \frac{w_{xg,yh,s}^{\theta}}{P^{\theta} + \sum_{zk \in \mathcal{Y}\times\mathcal{G}, t\in\mathcal{S}} w_{xg,zk,t}^{\theta}},$$

$$\Pr[u_{xg_i} = \varepsilon_{xg_i,0}] = \frac{P^{\theta}}{P^{\theta} + \sum_{zk \in \mathcal{Y}\times\mathcal{G}, t\in\mathcal{S}} w_{xg,zk,t}^{\theta}}.$$

Let $\mu_{xg,yh,s}^{supply} := \Pr\left[u_{xg_i} = \theta \ln \frac{w_{xg,yh,s}}{P} + \varepsilon_{xg_i,yh,s}\right] \times n_{xg}$ be the quantity of type xg workers who would like to supply for type yh entrepreneurs in sector s. Similarly, let $\mu_{xg,0} := \Pr[u_{xg_i} = \varepsilon_{xg_i,0}] \times n_{xg}$ be the quantity of type xg workers who would like to remain unmatched. Then, the supply by type xg workers for type yh entrepreneurs in sector s is given by,

$$\mu_{xg,yh,s}^{supply} = \mu_{xg,0} \times \left[\frac{w_{xg,yh,s}}{P}\right]^{\theta}. \tag{3}$$

By taking the log of both sides of the equation,

$$\ln \frac{\mu_{xg,yh,s}^{supply}}{\mu_{xg,0}} = \theta \ln \frac{w_{xg,yh,s}}{P}.$$

The parameter θ captures the labor supply elasticity. In particular, it measures the responsiveness of the extensive margin of the labor supply to real wages. Since labor supply choices are modeled as a binary decision (working vs. remaining unmatched), an adjustment mechanism in response to exogenous shocks in the model operates only through the extensive margin.[14]

Next, entrepreneur yh_j will maximize his (or her) indirect payoff by choosing a worker in a sector among $XGS + 1$ number of available matching alternatives:

$$v_{yh_j} = \max_{\substack{xg \in \mathcal{X}\times\mathcal{G} \\ s\in\mathcal{S}}} \{\theta \ln \frac{\pi_{xg,yh,s}}{P} + \eta_{xg,yh_j,s}, \eta_{0,yh_j}\}, \tag{4}$$

where $\pi_{xg,yh,s} := p_s q_{xg,yh,s} - w_{xg,yh,s}$. Similar to a worker's discrete choice problem, let $\mu_{xg,yh,s}^{demand} := \Pr\left[v_{yh_j} = \theta \ln \frac{\pi_{xg,yh,s}}{P} + \eta_{xg,yh_j,s}\right] \times m_{yh}$ be the quantity of type yh entrepreneurs who would like to demand for type xg workers in sector s. Likewise, let

[14] Heckman (1993) emphasized the importance of the distinction between the extensive and intensive margin of labor supply: labor supply choices at the extensive margin (i.e., labor-force participation and employment choices) and choices at the intensive margin (i.e., choices about hours of work or weeks of work for workers).

$\mu_{0,yh} := \Pr[v_{yh_j} = \eta_{0,yh_j}] \times m_{yh}$ be the quantity of type yh entrepreneurs who would like to remain unmatched. Then, the demand by type yh entrepreneurs for type xg workers in sector s is given by,

$$\mu^{demand}_{xg,yh,s} = \mu_{0,yh} \times \left[\frac{\pi_{xg,yh,s}}{P}\right]^{\theta}, \tag{5}$$

where $\pi_{xg,yh,s} := p_s q_{xg,yh,s} - w_{xg,yh,s}$.

3.3 Equilibrium

There are $X \times G \times Y \times G \times S$ labor matching markets for every combination of types of workers and entrepreneurs in every sector. Labor market clearing requires that supply by type xg workers for type yh entrepreneurs in sector s is equal to demand by type yh entrepreneurs for type xg workers in sector s for all matching markets, $\mu_{xg,yh,s} = \mu^{supply}_{xg,yh,s} = \mu^{demand}_{xg,yh,s}$ for all $xg \in \mathcal{X} \times \mathcal{G}$, $yh \in \mathcal{Y} \times \mathcal{G}$, and $s \in \mathcal{S}$.

Using equations (3) and (5), I can derive the following matching function: an equilibrium relationship between the quantity of matches between workers of type xg and entrepreneurs of type yh in sector s, $\mu_{xg,yh,s}$, and the quantity of unmatched workers of type xg, $\mu_{xg,0}$, the quantity of unmatched entrepreneurs of type yh, $\mu_{0,yh}$, and a price of good s, p_s:

$$\mu_{xg,yh,s} = \mu_{xg,yh,s}(\mu_{xg,0}, \mu_{0,yh}, p_s) = \left[\frac{p_s q_{xg,yh,s}}{P}\right]^{\theta} \times \left[\mu^{-1/\theta}_{xg,0} + \mu^{-1/\theta}_{0,yh}\right]^{-\theta}, \tag{6}$$

for all $xg \in \mathcal{X} \times \mathcal{G}$, $yh \in \mathcal{Y} \times \mathcal{G}$, and $s \in \mathcal{S}$.[15]

There are S goods markets. Because goods can freely move across countries and agents have the same systematic CES-type preference, the total value of demand for

[15]With Logit random utilities, it is well know that $\mu_{xg,yh,s}$ can be defined as a function of $\mu_{xg,0}$ and $\mu_{0,yh}$. Note also that the matching function $\mu_{xg,yh,s}(\mu_{xg,0}, \mu_{0,yh}, p_s)$ satisfies homogeneity of degree one in the quantity of unmatched workers and the quantity of unmatched entrepreneurs (constant returns to scale). Moreover, the above matching function provides a micro foundation for Mortensen and Pissarides (1994)-type homogeneous-of-degree-one matching function $m(u, v)$, where u and v represent the number of unemployed workers and vacancies respectively. Here, u (resp. v) corresponds to $\mu_{xg,0}$ (resp. $\mu_{0,yh}$). In addition, when $\theta = 1$, the matching function becomes the "Harmonic Matching Function" that has been widely used by demographers such as Schoen (1981).

good s is given by,

$$p_s q_s = \frac{p_s^{1-\sigma}}{P^{1-\sigma}} \sum_{\forall xg,yh,s} \mu_{xg,yh,s}(\mu_{xg,0}, \mu_{0,yh}, p_s) p_s q_{xg,yh,s}.$$

The total value of supply for good s is as follows:

$$\sum_{\substack{xg \in \mathcal{X} \times \mathcal{G} \\ yh \in \mathcal{Y} \times \mathcal{G}}} \mu_{xg,yh,s}(\mu_{xg,0}, \mu_{0,yh}, p_s) p_s q_{xg,yh,s}.$$

Goods market clearing condition requires that the total value of demand for good s is equal to the total value of supply for good s for all $s \in \mathcal{S}$.

Definition 2. *A matching function equilibrium is a solution of the following $XG+YG+S$ system of nonlinear equations with a triple $(\mu_{xg,0}, \mu_{0,yh}, p_s)$.*

$$\begin{cases} \mu_{xg,0} + \sum_{\forall yh,s} \mu_{xg,yh,s}(\mu_{xg,0}, \mu_{0,yh}, p_s) = n_{xg}, \quad \forall xg \in \mathcal{X} \times \mathcal{G}, \\ \mu_{0,yh} + \sum_{\forall xg,s} \mu_{xg,yh,s}(\mu_{xg,0}, \mu_{0,yh}, p_s) = m_{yh}, \quad \forall yh \in \mathcal{Y} \times \mathcal{G}, \\ \frac{p_s^{1-\sigma}}{P^{1-\sigma}} \sum_{\forall xg,yh,s} \mu_{xg,yh,s}(\mu_{xg,0}, \mu_{0,yh}, p_s) p_s q_{xg,yh,s} = \sum_{\forall xg,yh} \mu_{xg,yh,s}(\mu_{xg,0}, \mu_{0,yh}, p_s) p_s q_{xg,yh,s}, \ \forall s \in \mathcal{S}, \end{cases}$$

where $\mu_{xg,yh,s}(\mu_{xg,0}, \mu_{0,yh}, p_s) = \left[\frac{p_s q_{xg,yh,s}}{P}\right]^{\theta} \times \left[\mu_{xg,0}^{-1/\theta} + \mu_{0,yh}^{-1/\theta}\right]^{-\theta}$.

Let us characterize sharing rules in equilibrium. The sharing rules are specified by two vectors $w = (w_{xg,yh,s})_{xg \in \mathcal{X} \times \mathcal{G}, yh \in \mathcal{Y} \times \mathcal{G}, s \in \mathcal{S}}$ and $\pi = (\pi_{xg,yh,s})_{xg \in \mathcal{X} \times \mathcal{G}, yh \in \mathcal{Y} \times \mathcal{G}, s \in \mathcal{S}}$.[16] By plugging the matching function in equation (6) into the supply equation and the demand equation in (3) and (5), respectively, I can derive the following wage function and profit function:

$$\begin{aligned} w_{xg,yh,s} &= w_{xg,yh,s}(\mu_{xg,0}, \mu_{0,yh}, p_s) = \underbrace{p_s q_{xg,yh,s}}_{\text{Total surplus}} \underbrace{\frac{\mu_{xg,0}^{-1/\theta}}{\mu_{xg,0}^{-1/\theta} + \mu_{0,yh}^{-1/\theta}}}_{\text{Bargaining power}}, \\ \pi_{xg,yh,s} &= \pi_{xg,yh,s}(\mu_{xg,0}, \mu_{0,yh}, p_s) = \underbrace{p_s q_{xg,yh,s}}_{\text{Total surplus}} \underbrace{\frac{\mu_{0,yh}^{-1/\theta}}{\mu_{xg,0}^{-1/\theta} + \mu_{0,yh}^{-1/\theta}}}_{\text{Bargaining power}}, \end{aligned} \tag{7}$$

[16] Once a wage vector w is defined, then a profit vector π is automatically retrieved because $\pi_{xg,yh,s} := p_s q_{xg,yh,s} - w_{xg,yh,s}$.

for all $xg \in \mathcal{X} \times \mathcal{G}$, $yh \in \mathcal{Y} \times \mathcal{G}$, and $s \in \mathcal{S}$.[17]

The closed-form expressions of wage function and profit function above are new to this research arena. Admittedly, sharing rules can be easily recovered from Choo and Siow (2006)'s framework. However, the empirical matching literature has mainly focused on the identification of structural parameters from a matching function. This may be in part because transfers are not observed in marriage. To the best of my knowledge, Chan, Kroft and Mourifié (2019) provide a characterization of the equilibrium wage and matching functions simultaneously, like my framework. One key difference is that they assume that workers are perfect substitutes in their benchmark cases. Hence, the bargaining power mechanism is absent from wage determination in their framework. Later, they introduce a more general form of production function and characterize the wage function as an implicit equation.

The bargaining power of any match between a worker of type xg and an entrepreneur of type yh in sector s is determined endogenously by the quantity of unmatched workers of the same type $\mu_{xg,0}$ (supply) and the quantity of unmatched entrepreneurs of the same type $\mu_{0,yh}$ (demand). The endogenous sharing rule can be interpreted as a supply and demand framework. Suppose that there are more unmatched workers of type xg (supply) than unmatched entrepreneurs of type yh (demand), $\mu_{xg,0} > \mu_{0,yh}$, in sector s. In each two-person bargaining problem, the excess supply implies that there are relatively more outside alternatives for an entrepreneur of type yh and that there are relatively less outside alternatives for a worker of type xg.[18] Thus the excess supply increases the bargaining power of en-

[17] The above closed-form sharing rules have a nicer property than ones derived from the linear utility function a la Choo and Siow (2006). The wage and the profit are bounded between zero and total surplus. However, if $\theta = 1$ and the utility function takes the form of the linear utility function as in Choo and Siow (2006), then equilibrium sharing rules are characterized as follows:

$$w_{xg,yh,s} = w_{xg,yh,s}(\mu_{xg,0}, \mu_{0,yh}, p_s) = \frac{1}{2}\left[\underbrace{p_s q_{xg,yh,s}}_{\text{Total surplus}} + \underbrace{\ln \mu_{0,yh} - \ln \mu_{xg,0}}_{\text{Bargaining power}}\right],$$

$$\pi_{xg,yh,s} = \pi_{xg,yh,s}(\mu_{xg,0}, \mu_{0,yh}, p_s) = \frac{1}{2}\left[\underbrace{p_s q_{xg,yh,s}}_{\text{Total surplus}} + \underbrace{\ln \mu_{xg,0} - \ln \mu_{0,yh}}_{\text{Bargaining power}}\right]. \qquad (8)$$

In this case, the wage and the profit can take negative values or can have greater values than the total surplus.

[18] Since the surplus function $q_{xg,yh,s}$ does not depend on individual heterogeneity (i.e. i or j), any workers of the same type (or any entrepreneurs of the same type) are perfect substitutes in the production process. Hence, the unmatched number of workers are potential outside options

trepreneurs while it decreases the bargaining power of workers.

The structure of a sorting, matching, and sharing problem can explain endogenous sharing rules in equilibrium. Because each two-person bargaining problem is nested in a matching market and the matching market is also affected by other matching markets, the sharing rule for each two-person bargaining problem is determined by a system-wide network structure. Rubinstein and Wolinsky (1985) find that, in a random matching with a sequential bargaining process, bargaining power is determined by *the relative quantity of buyers and sellers* as players become infinitely patient. Manea (2011) extends the idea of Rubinstein and Wolinsky (1985) to a network structure and demonstrate that the shortage ratio of the mutually estranged set, defined as *the ratio of the number of partners to estranged players*, determines the collective bargaining power of its members. Similarly, the model's sharing rule depends on *the ratio of the number of unmatched workers to the number of unmatched entrepreneurs*, both of which are determined endogenously in equilibrium.

I can also express equilibrium in terms of an excess demand system.[19] In a labor matching market where a worker $xg \in \mathcal{X} \times \mathcal{G}$ and an entrepreneur $yh \in \mathcal{Y} \times \mathcal{G}$ are matched in sector $s \in \mathcal{S}$, workers are treated as suppliers and entrepreneurs as purchasers; a transfer $w_{xg,yh,s}$ as a price. An increase in $w_{xg,yh,s}$ raises the supply of workers while it decreases the demand for workers. In goods market s, an increase in p_s raises the supply of good s while it reduces the demand for good s.

Definition 3. *A competitive equilibrium is defined by a pair (w, p) at which the labor matching markets and goods markets clear, such that*

$$\begin{cases} \dfrac{\pi^{\theta}_{xg,yh,s}}{P^{\theta} + \sum_{zk \in \mathcal{X}\times\mathcal{G}, t\in\mathcal{S}} \pi^{\theta}_{zk,yh,t}} \times m_{yh} - \dfrac{w^{\theta}_{xg,yh,s}}{P^{\theta} + \sum_{zk\in\mathcal{Y}\times\mathcal{G},t\in\mathcal{S}} w^{\theta}_{xg,zk,t}} \times n_{xg} = 0, \ \forall xg, yh, s, \\ \dfrac{p_s^{1-\sigma}}{P^{1-\sigma}} \sum_{\forall xg,yh,s} \left[\dfrac{p_s q_{xg,yh,s}}{P}\right]^{\theta} \times \left[\mu_{xg,0}^{-1/\theta} + \mu_{0,yh}^{-1/\theta}\right]^{-\theta} p_s q_{xg,yh,s} \\ - \sum_{\forall xg,yh} \left[\dfrac{p_s q_{xg,yh,s}}{P}\right]^{\theta} \times \left[\mu_{xg,0}^{-1/\theta} + \mu_{0,yh}^{-1/\theta}\right]^{-\theta} p_s q_{xg,yh,s} = 0, \ \forall s, \end{cases}$$

where $\pi_{xg,yh,s} := p_s q_{xg,yh,s} - w_{xg,yh,s}$, $\mu_{xg,0} := \dfrac{P^{\theta}}{P^{\theta} + \sum_{zk\in\mathcal{Y}\times\mathcal{G},t\in\mathcal{S}} w^{\theta}_{xg,zk,t}}$, *and* $\mu_{0,yh} :=$

for entrepreneurs, and the unmatched number of entrepreneurs are potential outside options for workers.

[19] I follow Azevedo and Leshno (2016) and Galichon, Kominers and Weber (2019) to reformulate the equilibrium of the matching model as an excess demand system.

$$\frac{P^\theta}{P^\theta + \sum_{zk \in \mathcal{X} \times \mathcal{G}, t \in \mathcal{S}} \pi^\theta_{zk,yh,t}}.$$

3.4 Existence of an Equilibrium

Given a price vector $p = (p_s)$, a unique pairwise stable matching leads to a unique market-clearing wage vector $w = (w_{xg,yh,s})$, which follows from the matching literature (see Galichon, Kominers and Weber, 2019; Gayle and Shephard, 2019). Let $Z^L(w; p)$ be the excess demand function in the labor matching market clearing conditions given a price vector $p = (p_s)$. I can easily verify that the excess demand function Z^L satisfies the gross substitutability condition.[20] Galichon, Kominers and Weber (2019) proved that under the gross substitutes property and a given price vector, there exists a unique wage vector $w = (w_{xg,yh,s})$ such that $Z^L(w; p) = 0$.[21]

Similarly, given a wage vector $w = (w_{xg,yh,s})$, a unique market-clearing price vector $p = (p_s)$ exists by standard general equilibrium theory (see Proposition 17.F.3 of Mas-Colell, Whinston and Green, 1995, p 613). Let $Z^G(p; w)$ be the excess demand function in the goods market clearing conditions given a wage vector $w = (w_{xg,yh,s})$. The CES demand system along with $\sigma > 1$ guarantees that the excess demand function Z^G satisfies the gross subsitutability condition (see Example 17.F.2 of Mas-Colell et al., 1995, p 612).

Let $p \in \mathcal{P} \subset \mathbb{R}^{|\mathcal{S}|}_{++}$ where $\mathcal{P}$ is nonempty, closed, bounded, and convex and $w \in \mathcal{W} \subset \mathbb{R}^{|\mathcal{X}|\times|\mathcal{G}|+|\mathcal{Y}|\times|\mathcal{G}|+|\mathcal{S}|}$. Let $f : \mathcal{P} \to \mathcal{W}$ be the continuous function with $w = f(p)$, which follows from the uniqueness of a wage vector given a price vector. Since $\mathcal{P}$ is compact, under continuous f, $f(p)$ is compact in $\mathcal{W}$. Let $g : \mathcal{W} \to \mathcal{P}$ be the continuous function with $p = g(w)$, which follows from the uniqueness of a price vector given a wage vector. Since both f and g are continuous functions, $g \circ f$ is a continuous function on $\mathcal{P}$. By the Brouwer fixed point theorem, if $g \circ f$ is a continuous self-map on $\mathcal{P}$, then there exists a price vector $p \in \mathcal{P}$ such that $g(f(p)) = p$.

[20]Following Galichon, Kominers and Weber (2019), I can define gross substitutes property as follows. If $w_{xg,yh,s}$ increases and all other entries of w remains constant, then: (a.1) $Z^L_{xg,yh,s}(w)$ decreases, (a.2) $Z^L_{xg',yh',s}(w)$ increases if either $xg = xg'$ or $yh = yh'$ (but both equalities do not hold), (a.3) $Z^L_{xg',yh',s}(w)$ remains constant if $xg \neq xg'$ and $yh \neq yh'$. (b) For any $xg \in \mathcal{X} \times \mathcal{G}$ and $yh \in \mathcal{Y} \times \mathcal{G}$, the sum $\sum_{xg' \in \mathcal{X}\times\mathcal{G}, yh' \in \mathcal{Y}\times\mathcal{G}} Z^L_{xg',yh'}(w)$ is a decreasing function of $w_{xg,yh,s}$.

[21]There are other technical restrictions on the feasible bargaining set $\mathcal{F}_{xg,yh,s}$ and the distributions of the idiosyncratic terms to derive the existence and uniqueness of an equilibrium. Given a price vector $p = (p_s)$, the assumptions imposed in my model satisfy the other technical restrictions in Galichon, Kominers and Weber (2019).

Hence, the existence of an equilibrium pair (w, p) follows.

Establishing the uniqueness of an equilibrium is more difficult. Let $Z(w, p)$ be the system-wide excess demand system. The proof of the uniqueness of an equilibrium is equivalent to showing that there exists a unique pair (w, p) such that $Z(w, p) = 0$. If the excess demand function Z is inverse isotone, then the uniqueness of an equilibrium follows (Berry, Gandhi and Haile, 2013). However, wages influence goods market-clearing conditions and prices affect labor market-clearing conditions non-monotonically in the Jacobian matrix of Z. Hence, the established results of Berry, Gandhi and Haile (2013) are not directly applicable to my model.

Alternatively, the approach of Alvarez and Lucas (2007) may help establish the uniqueness of an equilibrium in my model where they established the existence and uniqueness of an equilibrium in the Eaton and Kortum (2002) model. An equilibrium is defined by a pair (w, p) where $w = (w_1, ..., w_n)$ denotes a wage vector and $p = (p_{11}, ..., p_{nn})$ denotes a price vector in the Eaton and Kortum (2002). Alvarez and Lucas (2007) established the uniqueness of an equilibrium as follows. First, they solve for price p_{mi} as a function of wage vector w and proves the uniqueness of $p_{mi}(w)$. Next, they represent a labor market clearing condition as an excess demand system Z where the excess demand is a function of the only w. They prove the uniqueness of an equilibrium by establishing that the excess demand Z has the gross substitute property. One may follow this approach and try to establish the uniqueness of an equilibrium in my model. However, at this stage, this task is beyond the scope of this paper and I leave it for future research.

Note also that the equilibrium matching μ is stable since all agents maximize their indirect payoffs by solving discrete choice problems. Thus, it satisfies both the *individual rationality* condition and the *pairwise stability* condition. Another interesting feature is that the concept of an equilibrium in this paper is related to Gale (1986a,b, 1987)'s earlier studies, in which he investigated a model of random matching and bargaining when the number of agents is large. Under certain conditions (e.g., agents do not discount the future), he showed that any perfect equilibrium of the bargaining game implements a Walras allocation of the exchange economy. It is obvious that the equilibrium in my model is a Walrasian equilibrium because there is a pair (w, p) such that each agent maximizes his or her indirect payoff when prices and all markets clear. Alternatively, the model can be interpreted as a sorting, matching, and bargaining framework where there is a large number of workers and entrepreneurs who form a production team in a sector and then bargain over a set of feasible utilities in a one-to-one matching. In a frictionless setting and with a given price vector p, I find a pairwise stable matching μ that ensures an equilibrium wage vector w (a bargaining solution). Therefore, pairwise stable matching implements a Walras allocation.

3.5 Welfare in Equilibrium

Suppose that economists are interested in how agents' welfare changes in response to exogenous shocks such as a change in the number of workers, a change in the number of entrepreneurs, or a change in the production function (including a reduction in cross-country matching costs). In equilibrium, each agent has a different level of indirect payoff because of the stochastic nature of preference heterogeneity. However, due to the additively separable feature of indirect payoffs (see equations (2) and (4)), the income part of the indirect payoffs is identical for all workers (or entrepreneurs) of the same type. Also, the preference heterogeneity is independent of the income part of the indirect payoffs. Therefore, I can define welfare metrics at the type-level.

The ex-ante expected indirect payoff for worker xg_i (resp. entrepreneur yh_j) before he observes his realizations of a vector of preference heterogeneity ε_{xg_i} (resp. η_{yh_j}) can be expressed as (see Appendix 6.2 for a detailed derivation):

$$\mathbb{E}\left[u_{xg_i}\right] = \mathbb{E}\left[\max_{\substack{yh\in\mathcal{Y}\times\mathcal{G}\\ s\in\mathcal{S}}}\{\theta\times\ln\frac{w_{xg,yh,s}}{P}+\varepsilon_{xg_i,yh,s},\varepsilon_{xg_i,0}\}\right] = \ln\left[1+\sum_{\forall yh,s}\left[\frac{w_{xg,yh,s}}{P}\right]^{\theta}\right] = \ln\frac{n_{xg}}{\mu_{xg,0}},$$

$$\mathbb{E}\left[v_{yh_j}\right] = \mathbb{E}\left[\max_{\substack{xg\in\mathcal{X}\times\mathcal{G}\\ s\in\mathcal{S}}}\{\theta\times\ln\frac{\pi_{xg,yh,s}}{P}+\eta_{xg,yh_j,s},\eta_{0,yh_j}\}\right] = \ln\left[1+\sum_{\forall xg,s}\left[\frac{\pi_{xg,yh,s}}{P}\right]^{\theta}\right] = \ln\frac{m_{yh}}{\mu_{0,yh}}.$$

The type-level welfare metric is defined as the log of the ratio of the quantity of workers of type xg (resp. entrepreneurs of type yh) relative to the quantity of unmatched xg workers (resp. unmatched yh entrepreneurs). Because the expected payoff of remaining unmatched is zero, i.e. $\mathbb{E}[\varepsilon_{xg_i,0}] = \mathbb{E}[\eta_{0,yh_j}] = 0$, the expected indirect payoff measures the agent's expected gains from participating in the matching market.[22]

[22] To better understand the welfare metrics in my paper, it is worth noting that the welfare in an

It is worth noting that the type-level welfare metric can also be expressed as a logsum formula, i.e., $\ln\left[1+\sum_{\forall yh,s}\left[\frac{w_{xg,yh,s}}{P}\right]^{\theta}\right]$ or $\ln\left[1+\sum_{\forall xg,s}\left[\frac{\pi_{xg,yh,s}}{P}\right]^{\theta}\right]$. The logsum formula derived here is similar to that of Small and Rosen (1981) where they extend the measurement of welfare changes to a discrete choice model.[23] Following Small and Rosen (1981), I use changes in ex-ante expected indirect payoffs $\mathbb{E}[u_{xg}]$ and $\mathbb{E}[v_{yh}]$ for all $xg \in \mathcal{X} \times \mathcal{G}$ and $yh \in \mathcal{Y} \times \mathcal{G}$ in response to changes in exogenous shocks as welfare changes for all agents.

The parameter θ plays a key role in measuring welfare in an equilibrium.[24] Suppose that the parameter $\theta \in (0,1)$. In this case, agents' indirect payoffs depend more on the preference heterogeneity than on the consumption part. Then, agents prefer an equal distribution of real wages (or real profits) in available matching markets to an unequal distribution of real wages (or real profits).[25] When the parameter $\theta = 1$, agents only care about the total sum of real wages (or real profits). For $\theta \in (1,\infty)$, agents prefer an unequal distribution of real wages (real profits). The welfare metric in the model is "inequality-adjusted" as in Galle, Rodriguez-Clare and Yi (2017), Jones and Klenow (2016), and Antras, De Gortari and Itskhoki (2017).

The type-level welfare metric can then be aggregated up to the country level, and up to the world level, respectively, as follows:

$$\mathcal{W}_g := \sum_{x\in\mathcal{X}} n_{xg}\,\mathbb{E}[u_{xg}] + \sum_{y\in\mathcal{Y}} m_{yg}\,\mathbb{E}[v_{yg}], \qquad \forall g \in \mathcal{G},$$
$$\mathcal{W} := \sum_{g\in\mathcal{G}} \mathcal{W}_g.$$

equilibrium is defined as an ex-ante-based notion. Suppose that all agents know their types and equilibrium pair (w,p) except for preference heterogeneity. Each agent calculates his or her expected indirect payoff based on the equilibrium pair (w,p), which corresponds to the type-level welfare metric.

[23]In Small and Rosen (1981), for the Logit case, the welfare change is evaluated as $-(1/\lambda)\left[\ln\sum_j \exp(W_j)\right]_{W_1^0}^{W_1^f}$ where λ denotes the marginal utility of income, W_j corresponds to the income part of the indirect payoff in our model, W_1^0 and W_1^f are defined as the value taken by W_1 at the initial and final prices, respectively. See Small and Rosen (1981) for more details.

[24]The parameter corresponds to the relative weight on the consumption part of the utility, and it captures the extensive margin elasticity of labor supply with respect to the real wage.

[25]For a worker of type xg, the distribution of real wages is specified by a vector $\left(\frac{w_{xg,yh,s}}{P}\right)_{yh\in\mathcal{Y}\times\mathcal{G},s\in\mathcal{S}}$; for an entrepreneur of type yh, the distribution of real profits is specified by a vector $\left(\frac{\pi_{xg,yh,s}}{P}\right)_{xg\in\mathcal{X}\times\mathcal{G},s\in\mathcal{S}}$.

Using the welfare metric in country g, $\mathcal{W}_g$, and in the world, $\mathcal{W}$, I can evaluate welfare changes from exogenous shocks in the model from the country's social planner's perspective and world social planner's perspective. Note that the world welfare metric defined in this paper is identical to the total indirect surplus of agents in Galichon, Kominers and Weber (2019)'s framework.

4 Simple Examples

Given the model's primitive quintuplet $(q, n, m, \theta, \sigma)$,[26] I can find an equilibrium triplet $(\mu_{xg,0}, \mu_{0,yh}, p_s)_{xg\in\mathcal{X}\times\mathcal{G}, yh\in\mathcal{Y}\times\mathcal{G}, s\in\mathcal{S}}$ in a matching function equilibrium and further characterize a matching μ, sharing rules (w, π), and welfare $(\mathbb{E}[u_{xg}], \mathbb{E}[v_{yh}], \mathcal{W}_g)_{xg\in\mathcal{X}\times\mathcal{G}, yh\in\mathcal{Y}\times\mathcal{G}, g\in\mathcal{G}}$. In this paper, I seek to answer the impact of changes in the production vector $q = (q_{xg,yh,s})$ on the equilibrium triplet $(\mu_{xg,0}, \mu_{0,yh}, p_s)$. One may answer that question numerically. A better approach would be to derive analytical results. However, deriving analytical results from the equilibrium is a difficult task even under typical two-sided marriage matching models (i.e., without the goods market) because it involves nonlinear fixed point problems.[27] Decker, Lieb, McCann and Stephens (2013) and Graham (2013) derived some comparative static results using the implicit function theorem in the perfectly transferable Choo and Siow (2006) model. Most of their comparative statics focused on changes in the supply of men or women, which corresponds to the supply of workers or entrepreneurs, (n, m), in this paper. Since I focus on changes in the production vector $q = (q_{xg,yh,s})$ instead of the population (n, m), a direct extension of those studies may not fit my case. On top of that, unlike Choo and Siow (2006)'s framework, my model features an imperfectly transferable utility with both labor matching markets and goods markets. Hence, deriving analytical results is even more challenging.

Instead of using the implicit function theorem technique, I impose some symmetric assumptions on (q, n, m) to derive new results that may provide some insights on matching patterns and welfare implications.[28] I illustrate two simple examples

[26] $q = (q_{xg,yh,s})_{xg\in\mathcal{X}\times\mathcal{G}, yh\in\mathcal{Y}\times\mathcal{G}, s\in\mathcal{S}}$, $n = (n_{xg})_{xg\in\mathcal{X}\times\mathcal{G}}$, and $m = (m_{yh})_{yh\in\mathcal{Y}\times\mathcal{G}}$.

[27] Graham (2013) noted that "[t]he general equilibrium nature of the Choo and Siow model makes a complete understanding of its economic properties difficult. In his Canadian Economics Association Presidential Address, Siow (2008) noted that (i) whether an equilibrium matching was globally unique was an open question and (ii) that the substitution patterns generated by the model were poorly understood."

[28] Admittedly, it would be better if one could derive comparative statics results without further

to which the model can be applied in the international economics literature. The first case is a reduction in sector-specific matching costs. The second case is an economic integration agreement such as a bilateral investment treaty (BIT) between two countries in a three-country framework. Last, I provide one (theoretical) exercise to illustrate that rising income inequality can be accompanied by strengthening negative assortative matching.

4.1 Reductions in Sector-Specific Matching Costs

Proposition 1. *Suppose that* $q_{xg,yh,s} = \frac{q}{\tau}\ \forall xg \in \mathcal{X} \times \mathcal{G}, yh \in \mathcal{Y} \times \mathcal{G}, s \in \mathcal{S}$ *and* $n_{xg} = m_{yh} = n\ \forall xg \in \mathcal{X} \times \mathcal{G}, yh \in \mathcal{Y} \times \mathcal{G}$. *If matching costs in sector* 1 *drop from* $\tau_{g,h,1} = \tau$ *to* $\tau'_{g,h,1} < \tau\ \forall g, h$, *then*

(i) The relative price of good 1, $\frac{p_1}{p_s}$, *decreases for all* $s \neq 1$;

(ii) The quantity of unmatched agents is reduced for all types of workers and entrepreneurs;

(iii) In sector 1, *the quantity of matching increases and real incomes rise; in other sectors, real incomes increase but the change in the quantity of matching is ambiguous.*

(iv) As $\sigma \to 1$ *or* $\theta \to 0$, *the quantity of matching in other sectors is more likely to increase.*

Proof. See Appendix 6.3. □

The ease of matching in sector 1 serves to advance technologies in sector 1. The boost in productivity induces more agents to engage in matching markets in sector 1. This positive effect in sector 1 spills over to other sectors. The equilibrium terms of trade changes, and thus the price of good 1 decreases relative to the price in other sectors. In other sectors, owing to increases in the relative prices, agents have a greater incentive to engage in production.

The productivity effect in sector 1 and the relative price effect in other sectors reduce the quantity of unmatched agents for all types of workers and entrepreneurs. All agents are better off when sector-specific matching costs are lower. In sector 1, the positive productivity effect outweighs the negative relative price effect; in other sectors, there is only a positive relative price effect. Hence, real wages and real profits increase for all types of workers and entrepreneurs.

Turning to matching patterns, I find that the quantity of matching in sector 1 always increases, while the changes in the quantity of matching in other sectors

assumptions, but at this stage, this is left for future research.

are ambiguous. This is because the net positive effect in sector 1 is greater than the positive relative price effect in other sectors. Interestingly, the change in the quantity of matching in other sectors depends on two parameters, θ and σ. The parameter θ measures the responsiveness of the extensive margin of labor supply in matching markets, and the parameter σ denotes the elasticity of substitution between goods in the goods market. When θ approaches infinity, the labor supply becomes perfectly elastic; when θ approaches 0, the labor supply becomes perfectly inelastic. When σ approaches infinity, goods are perfect substitutes; when σ approaches one, the production function takes a Cobb-Douglas form. Hence, as $\sigma \to 1$ or $\theta \to 0$, the number of matches in other sectors is more likely to increase.

The welfare implication of falling sector-specific matching costs is closely related to the task-based offshoring model of Grossman and Rossi-Hansberg (2008), who identify the productivity effect, the relative price effect, and the labor supply effect of offshoring in the source country. A reduction in the sector-specific matching costs in my model corresponds to a reduction in the cost of trading tasks. I also identify the productivity effect in sector 1, the relative price effect in all sectors, and the labor supply effect of a reduction in the quantity of unmatched agents.

While the key results in both models suggest the same welfare implications, my model differs on several dimensions from the offshoring model of Grossman and Rossi-Hansberg (2008). I relax the assumption of potential patterns of complementarity between tasks in their production technology, where there are no interactions between subsets of tasks; my model allows for any degree of substitution and complementary between worker types and entrepreneur types. While Grossman and Rossi-Hansberg (2008) focused on the source country, my model can accommodate an arbitrary number of countries. I use an identical multi-country framework in the example, but the analysis can be extended to the asymmetric multi-country case with numerical solutions. In addition, a typical offshoring framework, including the model of Grossman and Rossi-Hansberg (2008), is based on full employment, while my framework allows for unemployment and vacancies in an equilibrium. Therefore it is possible to investigate how a reduction in sector-specific matching costs can reduce unemployment and vacancies.

4.2 Third-Country Effects of an Economic Integration Agreement

Proposition 2. *Suppose that* $\mathcal{G} = \{1, 2, 3\}$*,* $q_{xg,yh,s} = \frac{q}{\tau}\ \forall xg \in \mathcal{X} \times \mathcal{G}, yh \in \mathcal{Y} \times \mathcal{G}, s \in \mathcal{S}$ *and* $n_{xg} = m_{yh} = n\ \forall xg \in \mathcal{X} \times \mathcal{G}, yh \in \mathcal{Y} \times \mathcal{G}$*. If cross-country matching costs between country* 1 *and country* 2 *drop from* $\tau_{1,2,s} = \tau_{2,1,s} = \tau$ *to* $(\tau_{1,2,s})' = (\tau_{2,1,s})' < \tau\ \forall s$*, then*

(i) In countries 1 and 2, the quantity of unmatched agents decreases for all types of workers and entrepreneurs. In country 3, the quantity of unmatched agents increases for all types of workers and entrepreneurs;

(ii) The quantity of cross-country matching between country 1 and country 2 and the quantity of within-country matching in country 3 increase. The quantity of matching in all other cases decreases;

(iii) In within-country matching, real incomes do not change. In cross-country matching between country 1 and country 2, real incomes increase. In cross-country matching between country 1 (or 2) and country 3, real incomes increase for agents in country 1 (or 2) while real incomes decrease for agents in country 3.

Proof. See Appendix 6.3. □

Reductions in bilateral matching costs, facilitated, for example, by a bilateral economic integration agreement between country 1 and country 2, can affect agents in country 3 through changes in bargaining power between agents in country 1 (or 2) and agents in country 3. A productivity increase in cross-country matching between countries 1 and 2 induces more agents in countries 1 and 2 to form production teams with all other matching markets. Moreover, the productivity effect reduces the quantity of unmatched agents in countries 1 and 2 because of the positive productivity effect. Hence, agents in countries 1 and 2 are better off.

The reductions in the quantity of unmatched agents in countries 1 and 2 spill over to unrelated agents in country 3. Since there are fewer available country-1 (or -2) agents, the bargaining power of country-3 agents diminishes in cross-country matching markets between country 1 (or 2) and country 3, which feeds a reduction in real wages (and real salaries) for country-3 agents. Some country-3 agents who used to match with country-1 (or -2) agents revert to country 3 or become unmatched, leaving country-3 agents worse off.

The extent to which country-3 agents are hurt depends on the quantity of matching in the initial equilibrium. In a case where there are no cross-country matching markets between country 1 (or 2) and country 3, country-3 agents are not affected by the formation of a bilateral economic integration agreement between countries 1 and 2. However, the more integrated country 3 is with country 1 (or 2), the larger the negative spillover effects will be.

Finally, I identify one notable result from this exercise. Country-3's domestic production — i.e., the quantity of within-country matching in country 3 — increases with an economic integration agreement between countries 1 and 2.[29] This effect is magnified if country 3 is more integrated with country 1 (or 2). The reason is that some country-3 agents who revert to country 3 from cross-country matching markets form production teams between country-3 workers and country-3 entrepreneurs.

The results of third-country effects of an economic integration agreement are reminiscent of the trade creation and trade diversion found by Viner (1950) where formation of a customs union or free-trade agreement would benefit exporters and consumers in the trading bloc while hurting exporters in non-member countries. My results are in line with Viner's insight. However, his analysis is based on the mechanism of international trade, while the third-country effect analyzed here relies on the mechanism of cross-country team formation (such as foreign direct investment and offshoring) under frictionless international trade flows.

4.3 Assortative Matching and Income Inequality Revisited

Proposition 3. *Suppose that $\mathcal{G} = \{1\}$, $\mathcal{S} = \{1\}$, $\mathcal{X} = \{H, L\}$, $\mathcal{Y} = \{H, L\}$, $q_{x,y} = q$ and $n_x = m_y = n \; \forall x \in \mathcal{X}, y \in \mathcal{Y}$.[30] If technologies complement skills (i.e., the production vector changes to $q_{H,H} > q_{H,L} = q_{L,H} = q > q_{L,L}$), then*

(i) The quantity of unmatched high-skilled agents decreases while the quantity of unmatched low-skilled agents increases;

(ii) Real incomes increase for high-skilled agents in both labor matching markets. It is ambiguous which real income will increase the most in the two labor matching markets; On the contrary, real incomes decrease for low-skilled agents in both labor matching markets. It is ambiguous which real income will decrease the most in the two labor matching markets;

[29] This phenomenon is dubbed as "reshoring" in the international trade literature. Reshoring refers to the process of returning the production of goods back to the company's original country. The Brexit, the withdrawal of the United Kingdom (country 3) from the European Union (country 1 and country 2), could be a similar example of this case. In the UK, there has been debate about whether Brexit could accelerate reshoring among UK firms that want to return production the UK to avoid tariffs and other barriers. My model can predict this possibility of reshoring caused by Brexit.

[30] Let H denote high-skilled and let L be low-skilled.

(iii) The quantity of high-high matching will be larger than the quantity of low-low matching;

(iv) There will be three possible matching patterns: random matching, stochastic positive assortative matching, and stochastic negative assortative matching.

Proof. See Appendix 6.3. □

Since my model features closed-form expressions of wage and matching functions, (w, μ), one can use the model to think about whether rising inequality can be attributed to an increase in positive assortative matching (see Card, Heining and Kline, 2013; Greenwood, Guner, Kocharkov and Santos, 2014; Dupuy and Weber, 2019). To answer this question, I assume away the concept of countries and sectors and introduce a two-skill model (i.e., high-skilled and low-skilled). Then, the model boils down to the 2 × 2 stochastic marriage framework such as Becker (1973, 1974). Let us start with a perfectly symmetric case where $q_{x,y} = q$ and $n_x = m_y = n \ \forall x \in \mathcal{X}, y \in \mathcal{Y}$. In the initial equilibrium, the quantities of matching and incomes in all labor matching markets are the same.

Suppose that productivity increases in the HH-matching market, while productivity decreases in the LL-matching market. Because of the symmetric assumption, income will rise in the HH-matching market, and more high-skilled agents are sorted into the HH-matching market. Conversely, low-skilled agents are more likely to move away from the LL-matching market because income will decrease in the LL-matching market. Hence, the quantity of unmatched high-skilled agents will decrease while the quantity of unmatched low-skilled agents will increase.

The changes in the quantity of unmatched high-skilled agents and the quantity of unmatched low-skilled agents spill over to unrelated cross-matching markets (i.e., the HL-matching market and the LH-matching market).[31] Since there are fewer available high-skilled agents and more available low-skilled agents, the bargaining power of high-skilled agents increases, which feeds an increase in incomes for high-skilled agents and a decrease in incomes for low-skilled agents in cross-matching markets. Hence, incomes for high-skilled agents in both labor matching markets increase while incomes for low-skilled agents in both labor matching markets de-

[31]Note that I assume that the productivity levels in the HL-matching market and the LH-matching market do not change.

crease.[32] Therefore the skill-biased technological change increases income inequality in the model.

Does rising income inequality occur in tandem with increasing positive assortative matching? Let $\mu_{x,y}$ be the quantity of matching between type x workers and type y entrepreneurs. Following Siow (2015), a log odds ratio is defined as follows:

$$\mathbb{L} := \ln \frac{\mu_{H,H}\mu_{L,L}}{\mu_{H,L}\mu_{L,H}}.$$

If $\mathbb{L} > 0$, then there is stochastic positive assortative matching. If $\mathbb{L} = 0$, then there is random matching. If $\mathbb{L} < 0$, then there is stochastic negative assortative matching. If a change in each component of a matching vector $\mu = (\mu_{x,y})$ can be identified, we can establish whether increasing inequality is accompanied by rising positive assortative matching. Due to the symmetric assumption combined with the increase in incomes in the HH-matching market (and the decrease in incomes in the LL-matching market), the quantity of matching in the HH-matching market will be larger than the quantity of matching in the LL-matching market (i.e., $\mu_{H,H} > \mu_{L,L}$). However, I could not establish further clear-cut analytical results and found that three possibilities can arise: a) $\mu_{H,H} > \mu_{H,L} = \mu_{L,H} > \mu_{L,L}$, b) $\mu_{H,L} = \mu_{L,H} > \mu_{H,H} > \mu_{L,L}$, and c) $\mu_{H,H} > \mu_{L,L} > \mu_{H,L} = \mu_{L,H}$. In case b), $\mathbb{L} < 0$. Therefore rising income inequality can be accompanied by increasing negative assortative matching.

To further illustrate the (stochastic) negative assortative matching case, a numerical simulation result is provided in Appendix 6.4.1.[33] One can easily check that Propositions 3-(i) through 3-(iii) hold, including rising inequality in the new equilibrium. The key mechanism that generates (stochastic) negative assortative matching is that an increase in incomes for high-skilled agents is larger in cross-matching markets ($6 > 5$). Similarly, a decrease in incomes for low-skilled agents is smaller in cross-matching markets ($2 > 1$). Therefore, both high-skilled agents and low-skilled agents are more likely to sort into cross-matching markets than self-matching markets. This counterintuitive result originates from a change of bargaining power in cross-matching markets. The bargaining power of high-skilled agents rises due to positive productivity shock in the HH-matching market; on the contrary, the bar-

[32]However, it is unclear which income will rise most (resp. decrease most) for high-skilled agents (resp. low-skilled agents), which will depend upon the magnitude of $q_{H,H}$ and $q_{L,L}$.

[33]Interested readers can find the case of (stochastic) positive assortative matching in Appendix 6.4.2.

gaining power of low-skilled agents drops due to a positive productivity shock in the LL-matching market. Interestingly, more low-skilled agents sort into cross-matching markets in the new equilibrium than in cross-matching markets in the initial equilibrium ($500 > 444$), even though income declines ($2 < 4$).

5 Conclusion

This study develops a multi-country, multi-sector, and multi-factor model of two-sided one-to-one matching where agents in different countries can form cross-country teams. Through the lens of a sorting, matching, and bargaining framework, I believe that the model addresses some interesting questions in the international economics literature. I derive three simple comparative statics: that a reduction in sector-specific matching costs can increase welfare for all agents without conflicts of interest, that a bilateral economic integration agreement can affect unrelated agents in a third country, and that rising income inequality can be accompanied by increasing negative assortative matching.

Future research could take several further steps based on this approach. First, it would be interesting to endogenize task choices such that agents can not only sort into sectors but also choose between a managerial position and a production position. Second, one could extend the approach to many-to-one matching problems in which an endogenous quantity of production workers work for a given manager (or firm) in a production team. In particular, Chan, Kroft and Mourifié (2019) proposed a many-to-one matching model of the labor market where several types of workers sort into a firm. It would be fruitful to extend Chan, Kroft and Mourifié (2019) to allow for cross-country matching to analyze the impact of offshoring and/or FDI or to allow for multiple sectors to analyze the impact of international trade. Last, it would be particularly interesting to take the model to the data. I acknowledge that the empirical relevance of the model has not been discussed in this paper, in part because the data available to researchers are limited. To estimate the model, employer-employee matched data featuring cross-country matching information (i.e., country-1 entrepreneurs residing in country-1 are paired up with country-2 workers residing in country-2) information is required. Once such a dataset is readily available to researchers, parameters of the costs of cross-country matching (e.g., costs of offshoring or FDI) could be estimated using the model's framework.

6 Appendix

6.1 Derivation of Conditional Choice Probability

$$\Pr\left[u_{xg_i} = \theta \ln \frac{w_{xg,yh,s}}{P} + \varepsilon_{xg_i,yh,s}\right] = \mathbb{E}\left[\prod_{\forall(zk,t)\neq(yh,s)} F(\varepsilon_{xg_i,yh,s} + \theta \ln \frac{w_{xg,yh,s}}{P} - \theta \ln \frac{w_{xg,zk,t}}{P})\right]$$

$$= \int_{-\infty}^{\infty} \exp\left[-\sum_{\forall(zk,t)\neq(yh,s)} \exp\left(-(\varepsilon+\gamma) - \theta \ln \frac{w_{xg,yh,s}}{P} + \theta \ln \frac{w_{xg,zk,t}}{P}\right)\right]$$

$$\times \exp\left[-(\varepsilon+\gamma) - \exp\left(-(\varepsilon+\gamma)\right)\right] d\varepsilon.$$

Let $\xi = 1 + \sum_{\forall(zk,t)\neq(yh,s)} \exp\left[-\theta \ln \frac{w_{xg,yh,s}}{P} + \theta \ln \frac{w_{xg,zk,t}}{P}\right]$.
Then, $\Pr\left[u_{xg_i} = \theta \ln \frac{w_{xg,yh,s}}{P} + \varepsilon_{xg_i,yh,s}\right]$ can be represented as follows:

$$\begin{aligned}
\Pr\left[u_{xg_i} = \theta \ln \frac{w_{xg,yh,s}}{P} + \varepsilon_{xg_i,yh,s}\right] &= \int_{-\infty}^{\infty} \exp\left[-(\varepsilon+\gamma) - \xi \exp\left(-(\varepsilon+\gamma)\right)\right] d\varepsilon \\
&= \left[\frac{\exp\left(-\xi \exp\left(-(\varepsilon+\gamma)\right)\right)}{\xi}\right]_{-\infty}^{\infty} \\
&= \frac{1}{\xi} \\
&= \frac{\left[\frac{w_{xg,yh,s}}{P}\right]^{\theta}}{\sum_{\forall(zk,t)} \left[\frac{w_{xg,zk,t}}{P}\right]^{\theta}}.
\end{aligned}$$

6.2 Derivation of Ex-Ante Expected Utility

$$\begin{aligned}
\mathbb{E}\left[u_{xg_i}\right] &= \mathbb{E}\left[\max_{\substack{yh\in\mathcal{Y}\times\mathcal{G}\\ s\in\mathcal{S}}}\{\theta\ln\frac{w_{xg,yh,s}}{P}+\varepsilon_{xg_i,yh,s},\varepsilon_{xg_i,0}\}\right]\\
&= \theta\ln\frac{w_{xg,yh,s}}{P}+\mathbb{E}\left[\varepsilon_{xg_i,yh,s}\;\middle|\;(yh,s)=\operatorname*{argmax}_{(yh,s)\in\mathcal{Y}\times\mathcal{G}\times\mathcal{S}}\{\theta\ln\frac{w_{xg,yh,s}}{P}+\varepsilon_{xg_i,yh,s},\varepsilon_{xg_i,0}\}\right]\\
&= \theta\ln\frac{w_{xg,yh,s}}{P}+\left(\Pr\left[u_{xg_i}=\theta\ln\frac{w_{xg,yh,s}}{P}+\varepsilon_{xg_i,yh,s}\right]\right)^{-1}\\
&\times\int_{-\infty}^{\infty}\varepsilon\exp\left[-(\varepsilon+\gamma)-\xi\exp\left(-(\varepsilon+\gamma)\right)\right]d\varepsilon\\
&= \theta\ln\frac{w_{xg,yh,s}}{P}+\xi\int_{-\infty}^{\infty}\varepsilon\exp\left[-(\varepsilon+\gamma)-\xi\exp\left(-(\varepsilon+\gamma)\right)\right]d\varepsilon\\
&= \theta\ln\frac{w_{xg,yh,s}}{P}-\gamma-\xi\int_{-\infty}^{\infty}-(\varepsilon+\gamma)\exp\left[-(\varepsilon+\gamma)-\xi\exp\left(-(\varepsilon+\gamma)\right)\right]d\varepsilon.
\end{aligned}$$

Let $\Delta=-(\varepsilon+\gamma)$. Then, $\mathbb{E}\left[u_{xg_i}\right]$ can be represented as follows:

$$\begin{aligned}
\mathbb{E}\left[u_{xg_i}\right] &= \theta\ln\frac{w_{xg,yh,s}}{P}-\gamma-\xi\int_{-\infty}^{\infty}-(\varepsilon+\gamma)\exp\left[-(\varepsilon+\gamma)-\xi\exp\left(-(\varepsilon+\gamma)\right)\right]d\varepsilon\\
&= \theta\ln\frac{w_{xg,yh,s}}{P}-\gamma-\xi\int_{-\infty}^{\infty}\Delta\exp\left[\Delta-\xi\exp\left(\Delta\right)\right]d\Delta.
\end{aligned}$$

Using $\int_{-\infty}^{\infty}\Delta\exp\left[\Delta-\xi\exp\left(\Delta\right)\right]d\Delta=-\frac{\gamma+\ln\xi}{\xi}$,

$$\mathbb{E}\left[u_{xg_i}\right]=\theta\ln\frac{w_{xg,yh,s}}{P}+\ln\xi=\ln\left[1+\sum_{\forall yh,s}\left[\frac{w_{xg,yh,s}}{P}\right]^{\theta}\right].$$

Using the supply equation (3),

$$\mathbb{E}\left[u_{xg_i}\right]=\ln\left[1+\sum_{\forall yh,s}\frac{\mu_{xg,yh,s}}{\mu_{xg,0}}\right]=\ln\frac{n_{xg}}{\mu_{xg,0}}.$$

6.3 Proofs

6.3.1 Proof of Proposition 1

Proof. (i) The demand and supply for good s can be expressed as follows:

$$\frac{p_s^{-\sigma}}{P^{1-\sigma}}E = \sum_{\forall xg,yh} \mu_{xg,yh,s}(\mu_{xg,0}, \mu_{0,yh}, p_s) q_{xg,yh,s} = \sum_{\forall g,h} \left[\frac{p_s \frac{q}{\tau_{g,h,s}}}{P} \right]^{\theta} \times \frac{\mu_0}{2^\theta} \times \frac{q}{\tau_{g,h,s}}.$$

Hence, the relative price of good 1 can be represented as:

$$\left[\frac{p_1}{p_s}\right]^{-\sigma-\theta} = \frac{\sum_{\forall g,h} \tau_{g,h,1}^{-1-\theta}}{\sum_{\forall g,h} \tau_{g,h,s}^{-1-\theta}}. \tag{9}$$

Because matching costs in sector 1 drop from $\tau_{g,h,1} = \tau$ to $\tau'_{g,h,1} < \tau \ \forall g, h$, the relative price of good 1, $\frac{p_1}{p_s}$, decreases for all $s \neq 1$.

(ii) The labor market clearing condition for workers of type-xg can be expressed as follows:

$$\mu_0 + \sum_{\forall yh} \mu_{xg,yh,1}(\mu_0) + \sum_{\forall yh, s\neq 1} \mu_{xg,yh,s}(\mu_0) = n$$

$$\iff \mu_0 + \sum_{\forall h} \left[\frac{p_1 \frac{q}{\tau_{g,h,1}}}{P} \right]^{\theta} \frac{\mu_0}{2^\theta} + \sum_{\forall h, s\neq 1} \left[\frac{p_s \frac{q}{\tau_{g,h,s}}}{P} \right]^{\theta} \frac{\mu_0}{2^\theta} = n \tag{10}$$

Suppose that the quantity of unmatched agents weakly increases such that $\mu_0' \geq \mu_0$. From the previous proof, we know that $\frac{p_s'}{P'\tau'_{g,h,s}} > \frac{p_s}{P\tau_{g,h,s}}$. Hence it must be that $\frac{p_1'}{P'\tau'_{g,h,1}} < \frac{p_1}{P\tau_{g,h,1}} \ \forall g, h$ to satisfy equation (10) to hold.

However, this is a contradiction. From equation (9), we can derive

$$\left[\frac{p_1}{p_s}\right]^{-\sigma-\theta} = \left[\frac{\tau_{g,h,1}}{\tau_{g,h,s}}\right]^{-1-\theta}.$$

Plugging this equation into $\frac{p_1}{p_s}\frac{\tau_{g,h,s}}{\tau_{g,h,1}}$,

$$\frac{p_1}{p_s}\frac{\tau_{g,h,s}}{\tau_{g,h,1}} = \left[\frac{p_s}{p_1}\right]^{\frac{\sigma-1}{1+\theta}}.$$

Because the relative price of good 1 decreases, $\frac{p_1}{p_s}\frac{\tau_{g,h,s}}{\tau_{g,h,1}}$ should increase, which contradicts $\frac{p_1^{'}}{P'\tau_{g,h,1}^{'}} < \frac{p_1}{P\tau_{g,h,1}}$ $\forall g, h$. Hence, the quantity of unmatched agents, μ_0, drops for all types of workers and entrepreneurs.

(iii) In sector 1, the quantity of matching and real wages (or real profits) can be expressed as follows. For all g, h,

$$\mu_{xg,yh,1} = \left[\frac{p_1\frac{q}{\tau_{g,h,1}}}{P}\right]^{\theta} \times \frac{\mu_0}{2^{\theta}} \quad \text{and} \quad \frac{w_{xg,yh,1}}{P} = \frac{\pi_{xg,yh,1}}{P} = \frac{1}{2}\frac{p_1}{P}\frac{q}{\tau_{g,h,1}}.$$

In other sectors, the quantity of matching and real wages (or real profits) can be expressed as follows. For all $s \neq 1$,

$$\mu_{xg,yh,s} = \left[\frac{p_s\frac{q}{\tau_{g,h,s}}}{P}\right]^{\theta} \times \frac{\mu_0}{2^{\theta}} \quad \text{and} \quad \frac{w_{xg,yh,s}}{P} = \frac{\pi_{xg,yh,s}}{P} = \frac{1}{2}\frac{p_s}{P}\frac{q}{\tau_{g,h,s}}.$$

Since $\frac{p_s^{'}}{P'\tau_{g,h,s}^{'}} > \frac{p_s}{P\tau_{g,h,s}}$ $\forall g, h, s$, it is obvious that real wages and real profits increase for all sectors s.

From the labor market clear condition in equation (10), it must be that: a) $\mu_{xg,yh,1}$ increases and $\mu_{xg,yh,s}$ decreases, b) $\mu_{xg,yh,1}$ decreases and $\mu_{xg,yh,s}$ increases, and c) both $\mu_{xg,yh,1}$ and $\mu_{xg,yh,s}$ increase. We can express the relative ratio of the quantity of matching in sector 1 to the quantity of matching in sector s as follows:

$$\frac{\mu_{xg,yh,1}}{\mu_{xg,yh,s}} = \left[\frac{p_1}{p_s}\frac{\tau_{g,h,s}}{\tau_{g,h,1}}\right]^{\theta} = \left[\left[\frac{p_s}{p_1}\right]^{\frac{\sigma-1}{1+\theta}}\right]^{\theta}. \tag{11}$$

Because the relative price of good 1, $\frac{p_1}{p_s}$, decreases for all $s \neq 1$, the relative ratio of the quantity of matching in sector 1 to the quantity of matching in sector s,

$\frac{\mu_{xg,yh,1}}{\mu_{xg,yh,s}}$ increases. This implies that the case b) is ruled out. Therefore, the quantity of matching in sector 1, $\mu_{xg,yh,1}$, should increase. However, a change in the quantity of matching in other sectors is ambiguous.

(iv) In equation (11), the relative ratio of the quantity of matching in sector 1 to the quantity of matching in sector s is an increasing function of σ and θ. Also, the relative ratio is equal to one when $\sigma \to 1$ or $\theta \to 0$. From the previous proof, $\mu_{xg,yh,1}$ always increases. Therefore, $\mu_{xg,yh,1}$ is more likely to increase as $\sigma \to 1$ or $\theta \to 0$.

□

6.3.2 Proof of Proposition 2

Proof. (i) Let μ_{0A} be the quantity of unmatched agents in country 1 or 2 and μ_{0B} be the quantity of unmatched agents in country 3. The labor market clearing conditions can be expressed as follows:

$$\mu_{0A} + \sum_{y=1,h,s} \mu_{xg,yh,s}(\mu_{0A}) + \sum_{y=2,h,s} \mu_{xg,yh,s}(\mu_{0A}) + \sum_{y=3,h,s} \mu_{xg,yh,s}(\mu_{0A}, \mu_{0B}) = n \qquad (12)$$

$$\mu_{0B} + \sum_{y=1,h,s} \mu_{xg,yh,s}(\mu_{0A}, \mu_{0B}) + \sum_{y=2,h,s} \mu_{xg,yh,s}(\mu_{0A}, \mu_{0B}) + \sum_{y=3,h,s} \mu_{xg,yh,s}(\mu_{0B}) = n \qquad (13)$$

Since $\tau_{1,2,s} = \tau_{2,1,s} = \tau$ to $\tau'_{1,2,s} = \tau'_{2,1,s} < \tau \ \forall s$, the labor market condition in equation (12) dictates that either $\mu_{0A} < \mu'_{0A}$ or $\mu_{0B} < \mu'_{0B}$. Suppose that $\mu_{0A} < \mu'_{0A}$. Then, it must be that $\mu_{xg,yh,s}(\mu_{0A}, \mu_{0B}) > \mu_{xg,yh,s}(\mu'_{0A}, \mu'_{0B})$, which implies that $\mu_{0B} > \mu'_{0B}$. However, this contradicts equation (13). Hence, $\mu_{0A} > \mu'_{0A}$ and $\mu_{0B} < \mu'_{0B}$.

(ii) Since $\mu_{0B} < \mu'_{0B}$, it must be that $\mu_{xg,yh,s}(\mu_{0B}) < \mu_{xg,yh,s}(\mu'_{0B})$. Both inequalities imply that $\mu_{xg,yh,s}(\mu_{0A}, \mu_{0B}) > \mu_{xg,yh,s}(\mu'_{0A}, \mu'_{0B})$. In countries 1 and 2, since $\mu_{0A} > \mu'_{0A}$ the quantity of within-country matching should decrease $\mu_{xg,yh,s}(\mu_{0A}) > \mu_{xg,yh,s}(\mu'_{0A})$ while the quantity of cross-country matching between country 1 and 2 should increase $\mu_{xg,yh,s}(\mu_{0A}) < \mu_{xg,yh,s}(\mu'_{0A})$.

(iii) Because within-country matching costs do not change and workers and entrepreneurs are symmetric, real wages (and real profits) do not change in within-country matching:

$$\frac{w_{xg,yg,s}}{P} = \frac{\pi_{xg,yg,s}}{P} = \frac{1}{2}\frac{p_s}{P}\frac{q}{\tau}.$$

Since $\tau_{1,2,s} = \tau_{2,1,s} = \tau$ to $\tau'_{1,2,s} = \tau'_{2,1,s} < \tau \ \forall s$ and the symmetry between workers and entrepreneurs, real wages (and real profits) increase in cross-country matching between country 1 and 2:

$$\frac{w_{x1,y2,s}}{P} = \frac{\pi_{x1,y2,s}}{P} = \frac{1}{2}\frac{p_s}{P}\frac{q}{\tau_{1,2,s}} \quad \text{and} \quad \frac{w_{x2,y1,s}}{P} = \frac{\pi_{x2,y1,s}}{P} = \frac{1}{2}\frac{p_s}{P}\frac{q}{\tau_{2,1,s}}.$$

Because $\mu_{0A} > \mu_{0A}^{'}$ and $\mu_{0B} < \mu_{0B}^{'}$, real incomes increase for agents in country 1 and 2 and real incomes decrease for agents in country 3 in cross-country matching between country 1 (or 2) and 3.

$$\frac{w_{x1,y3,s}}{P} = \frac{w_{x2,y3,s}}{P} = \frac{\pi_{x3,y1,s}}{P} = \frac{\pi_{x3,y2,s}}{P} = \frac{p_s}{P}\frac{q}{\tau}\frac{\mu_{0A}^{-1/\theta}}{\mu_{0A}^{-1/\theta} + \mu_{0B}^{-1/\theta}},$$

$$\frac{w_{x3,y1,s}}{P} = \frac{w_{x3,y2,s}}{P} = \frac{\pi_{x1,y3,s}}{P} = \frac{\pi_{x2,y3,s}}{P} = \frac{p_s}{P}\frac{q}{\tau}\frac{\mu_{0B}^{-1/\theta}}{\mu_{0A}^{-1/\theta} + \mu_{0B}^{-1/\theta}}.$$

□

6.3.3 Proof of Proposition 3

Proof. (i) Let μ_{0H} be the quantity of unmatched high-skilled agents and μ_{0L} be the quantity of unmatched low-skilled agents. The labor market clearing conditions can be expressed as follows:

$$\mu_{0H} + \left[\frac{q_{H,H}}{2}\right]^{\theta}\mu_{0H} + q^{\theta}\left[\mu_{0H}^{-1/\theta} + \mu_{0L}^{-1/\theta}\right]^{-\theta} = n, \tag{14}$$

$$\mu_{0L} + \left[\frac{q_{L,L}}{2}\right]^{\theta}\mu_{0L} + q^{\theta}\left[\mu_{0H}^{-1/\theta} + \mu_{0L}^{-1/\theta}\right]^{-\theta} = n. \tag{15}$$

Combining equations (14) and (15), we can express the relative ratio of the quantity of unmatched high-skilled agents to the quantity of unmatched low-skilled agents as follows:

$$\frac{\mu_{0H}}{\mu_{0L}} = \frac{1 + \left[\frac{q_{L,L}}{2}\right]^{\theta}}{1 + \left[\frac{q_{H,H}}{2}\right]^{\theta}}. \tag{16}$$

Since $q_{H,H} > q_{L,L}$, it must be that $\mu_{0H} < \mu_{0L}$. Let μ_{00} be the quantity of unmatched agents before the technical change. The labor market clearing condition before the technical change can be expressed as follows:

$$\mu_{00} + \left[\frac{q}{2}\right]^{\theta} \mu_{00} + \left[\frac{q}{2}\right]^{\theta} \mu_{00} = n. \tag{17}$$

Suppose that $\mu_{0L} > \mu_{0H} > \mu_{00}$. Since $q_{H,H} > q$, the labor market clearing condition in equation (14) does not hold. Suppose that $\mu_{00} > \mu_{0L} > \mu_{0H}$. Since $q > q_{L,L}$, the labor market clearing condition in equation (15) does not hold. Hence, it must be that $\mu_{0L} > \mu_{00} > \mu_{0H}$.

(ii) Because workers and entrepreneurs are symmetric, real incomes for high-skilled agents can be expressed as follows in the HH matching market and in the HL (or LH) matching market respectively:

$$w_{H,H} = \pi_{H,H} = \frac{1}{2} q_{H,H}, \qquad w_{H,L} = \pi_{L,H} = q \frac{\mu_{0H}^{-1/\theta}}{\mu_{0H}^{-1/\theta} + \mu_{0L}^{-1/\theta}}.$$

Since $q_{H,H} > q$ and $\mu_{0H} < \mu_{0L}$, real incomes for high-skilled agents must increase in the both labor matching markets. However, it is ambiguous which real income will increase more.

Similarly, real incomes for low-skilled agents can be expressed as follows in the LL matching market and in the LH (or HL) matching market respectively:

$$w_{L,L} = \pi_{L,L} = \frac{1}{2} q_{L,L}, \qquad w_{L,H} = \pi_{H,L} = q \frac{\mu_{0L}^{-1/\theta}}{\mu_{0H}^{-1/\theta} + \mu_{0L}^{-1/\theta}}.$$

Since $q_{L,L} < q$ and $\mu_{0H} < \mu_{0L}$, real incomes for low-skilled agents must decrease in the both labor matching markets. However, it is ambiguous which real income will decrease more.

(iii) Because $\mu_{0L} > \mu_{0H}$ and the labor matching market clearing conditions in equation (14) and (15), it must be that $\mu_{H,H} > \mu_{L,L}$.

(iv) Please refer to numerical simulation in Appendix 6.4.

□

6.4 Numerical Simulation of Proposition 3

6.4.1 Stochastic Negative Assortative Matching

- Assume that $(n_H, n_L, m_H, m_L, \theta) = (1000, 1000, 1000, 1000, 1)$ and surplus $(q_{x,y})$ is give by:

$x\backslash y$	H	L		$x\backslash y$	H	L
H	8	8	$\Rightarrow$	H	10	8
L	8	8		L	8	2

- Matching $(\mu_{x,y})$:

$x\backslash y$	0	H	L			$x\backslash y$	0	H	L	
0		111	111			0		83	250	
H	111	444	444	1000	$\Rightarrow$	H	83	417	500	1000
L	111	444	444	1000		L	250	500	250	1000
		1000	1000					1000	1000	

- Wages $(w_{x,y})$:

$x\backslash y$	H	L		$x\backslash y$	H	L
H	4	4	$\Rightarrow$	H	5	6
L	4	4		L	2	1

- Profits $(\pi_{x,y})$:

$x\backslash y$	H	L		$x\backslash y$	H	L
H	4	4	$\Rightarrow$	H	5	2
L	4	4		L	6	1

6.4.2 Stochastic Positive Assortative Matching

- Assume that $(n_H, n_L, m_H, m_L, \theta) = (1000, 1000, 1000, 1000, 1)$ and surplus $(q_{x,y})$ is give by:

$x \backslash y$	H	L		$x \backslash y$	H	L
H	8	8	$\Rightarrow$	H	16	8
L	8	8		L	8	6

- Matching $(\mu_{x,y})$:

$x \backslash y$	0	H	L			$x \backslash y$	0	H	L	
0		111	111			0		69	155	
H	111	444	444	1000	$\Rightarrow$	H	69	550	381	1000
L	111	444	444	1000		L	155	381	464	1000
		1000	1000					1000	1000	

- Wages $(w_{x,y})$:

$x \backslash y$	H	L		$x \backslash y$	H	L
H	4	4	$\Rightarrow$	H	8	5.5
L	4	4		L	2.5	3

- Profits $(\pi_{x,y})$:

$x \backslash y$	H	L		$x \backslash y$	H	L
H	4	4	$\Rightarrow$	H	8	2.5
L	4	4		L	5.5	3

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