Two-Sided Heterogeneity, Endogenous Bargaining, and International Matching Markets

Jaerim Choi*
University of Hawaii at Manoa

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Abstract

This paper develops a multi-country, multi-sector, and multi-factor with a two-sided one-to-one matching model. The sharing rule of each two-person bargaining problem is endogenously determined in each matching market that is linked to an interdependent network structure of a world economy, which breaks away from competitive marginal productivity theories of factor returns. Using this new theoretical framework, we study how falling costs of offshoring affects factor prices and how bilateral economic integration agreement affects a third-country. We demonstrate that offshoring can increase welfare for all agents without conflict of interests, and bilateral economic integration agreement can have effects on the third-country through the bargaining channel.

Keywords: Two-Sided Heterogeneity; Bargaining; Matching.
JEL Code: F23, F66, C78, D33, J23, J31

*Department of Economics, University of Hawaii at Manoa, Saunders Hall 542, 2424 Maile Way, Honolulu, HI 96822. Phone: (808) 956 - 7296. E-mail: choijm@hawaii.edu. Website: www.jaerimchoi.com.

1 Introduction

A two-sided matching model is a burgeoning interest in international trade literature (Eaton, Jinkins, Tybout and Xu, 2016; Bernard, Moxnes and Ulltveit-Moe, 2018). When heterogeneous firms find a new matching partner to sell or buy goods in foreign markets, they must incur a search cost or a relationship-specific fixed cost to match with each partner. Most current studies of two-sided matching models in international trade literature deal with this exporter-importer matching problem (international goods market). Likewise, it is also a vital research arena to understand why entrepreneurs hire foreign workers to produce, who matches whom, and its distributional consequences on both heterogeneous entrepreneurs and heterogeneous workers (international factor market) as foreign direct investment around the world has increased tremendously with rapid advancement in transportation and communication technology. However, there have been far fewer studies on a two-sided international matching of heterogeneous entrepreneurs and heterogeneous workers (Kremer and Maskin, 2006; Antràs, Garicano and Rossi-Hansberg, 2006; Choi, 2018). Moreover, the existing studies that analyze the international two-sided matching problem of factor market are based only on a two-country framework and investigate a move from Autarky to (complete) Globalization because adding a multi-dimension (> 2) into the model complicates the problem. Then, what are the distributional impacts of country A and country B signing a bilateral investment treaty on agents in country C?

In this paper, we fill this gap in the literature by developing a multi-country, multi-sector, and multi-factor with two-sided heterogeneity model with one-to-one matching between heterogeneous entrepreneurs and heterogeneous workers. To overcome the methodological challenge, we borrow this paper's framework from both two-sided matching literature and international trade literature. Specifically, we extend a Galichon, Kominers and Weber (2018)'s two-sided matching model to allow for multiple countries and multiple sectors as in Costinot (2009). By doing so, we can answer questions in international economics using the tools and techniques developed in the two-sided matching literature. From an international trade literature perspective, we extend a multi-country, multi-sector, and multi-factor neoclassical trade model of Costinot (2009) to allow for complementarity effect between factors and add to it a matching and bargaining problem.

The notable feature in this paper is that the sharing rule in each one-to-one matching is endogenously determined, which breaks away from competitive marginal productivity theories of factor returns. More specifically, in a matching market between country g - type x workers and country h - type y entrepreneurs in sector s, two sufficient statistics determines the sharing rule: the number of unmatched country g - type x workers and the number of unmatched country h - type y entrepreneurs. The sharing rule is similar to that of Rubinstein and Wolinsky (1985) in which they find that, in a random matching with a sequential bargaining process, bargaining power is determined by the relative size of buyers and sellers as players become infinitely patient. However, unlike Rubinstein and Wolinsky (1985), the relative size of unmatched workers and unmatched entrepreneurs is endogenously determined. The endogenous sharing rule can also be interpreted as a supply and demand framework. As the number of the unmatched country g - type x workers (supply) rises compared to the number of the unmatched country h - type y entrepreneurs (demand), outside options for the entrepreneurs compared to the workers increase, which raises bargaining powers for entrepreneurs. Using this novel framework, we study how globalization affects the bargaining power of each agent that feeds through into factor returns.

The other noticeable characteristic in the model is that we can measure the welfare of each agent by calculating the expected utility of ex-ante identical agents. Behind the veil of ignorance (Rawls, 1971), a worker in the country g - type x worker (or an entrepreneur in the country h - type y) is uncertain about his matching partner including the case of remaining single. We derive ex-ante expected utility for country g - type x workers and expected utility for country h - type y entrepreneurs, respectively, and demonstrate that the formula is expressed as a logsum which is identical to the welfare formula of Small and Rosen (1981) where they study measurement of welfare changes in the discrete choice model. The group-level welfare metrics are then aggregated up to the country-level and again aggregated up to the world-level. In addition, the social welfare formula, calculated from summation of ex-ante expected utilities, is "inequality-adjusted" as in Jones and Klenow (2016) in macroeconomics literature and Galle, Rodriguez-Clare and Yi (2017) and Antras, De Gortari and Itskhoki (2017) in international trade literature. With the social welfare formula in our model, we conduct a systematic welfare analysis on the impact of globalization: who benefits and who loses from globalization?

We present two comparative statics predictions that shed some light on consequences of globalization. First, we explore the impact of the reduction in sector-specific offshoring cost. Contrary to existing studies on offshoring that generate a conflict of interests between agents, all agents can benefit from the offshoring process in every country. Suppose that a cross-country matching cost in sector s decreases. In sector s, the cost reduction boosts productivities which increase total surplus in each cross-country matching. In other sectors, an equilibrium terms of trade changes and the relative price increases, which also increases total surplus in each matching. The only exception to this case is the within-country matching market in sector s because the relative price decreases and the productivity does not change. However, agents in the within-country matching market in sector s adjust to the negative impact by finding partners in the cross-country matching market in sector s or other sectors. Thus, their combined effects reduce the number of unmatched agents for all types of workers and entrepreneur, which raises welfare for all agents.

Next, we study the implications of bilateral economic integration such as country A and country B signing a bilateral investment treaty on agents in country C. The welfare for agents in country C diminishes from this bilateral investment treaty because the bargaining powers of agents in country C decline in the cross-country matching markets such as a matching market between agents in country A (or country B) and agents in country C. The reduction in bargaining power for agents in country C lowers the share of total surplus that goes to agents in country C. Interestingly, the number of within-country matches rises in country C because some agents in country C who once matched with agents in country A or B revert to country C (a phenomenon of reshoring) and re-match within the country C.

2 Related Literature

Offshoring. This paper is closely related to Kremer and Maskin (2006), Antràs, Garicano and Rossi-Hansberg (2006), and Choi (2018) where they modeled globalization of production process as a cross-country matching between agents and study the distributional impact of globalization in each country. However, this paper is novel in several dimensions. First, the modeling framework is extended to a multi-country setting while the previous models are limited to a two-country framework. The multi-country framework enables us to analyze the impact of falling costs of matching between two countries on unrelated third-country parties, a network effect. Second, we explicitly incorporate a cross-country matching cost into the model. The existing studies only consider a move from completely closed two economies to a perfectly integrated international economy. Third, the sharing process in each one-to-one matching is modeled as a bargaining problem, and the sharing rule is endogenously determined in the model.

Two-Sided Matching. Another related research area is a two-sided one-to-one marriage matching market literature. Choo and Siow (2006) propose a stochastic version of Becker (1973, 1974)'s classic static transferable utility model of the marriage markets by incorporating random identically distributed McFadden (1974)-type noise in the preferences of each of the participants. Recently, Galichon, Kominers and Weber (2018) provide a general framework of imperfectly transferable utility and unobserved heterogeneity in tastes. This paper extends Galichon, Kominers and Weber (2018)'s framework to allow for multiple countries. Instead of applying to the marriage market, we apply their framework in the context of international factor matching markets in which heterogeneous entrepreneurs hire heterogeneous workers to produce a good. The model's aggregate matching function is related to Mortensen and Pissarides (1994)-type constant returns to scale reduced-form matching function. Unlike Mortensen and Pissarides (1994), our model endogenously derives aggregate matching function with constant returns to scale by the market clearing condition.

Bargaining. The paper is also related to the standard two-person Nash (1950) bargaining problem. In each one-to-one matching, a worker and an entrepreneur have to divide the total surplus into two parts. In our model, the two-person bargaining problem is nested in a matching market of the same type-country worker and the same type-country entrepreneur, which is also nested in an entire economic system. We show that the bargaining solution is endogenously determined in the model and the bargaining power can be expressed as a function of the mass of unemployed workers of the same type-country (supply) and the mass of unmatched entrepreneurs of the same type-country (demand) in each matching market. We demonstrate that the bargaining solution in our model becomes identical to the axiomatic solution in Nash (1950) only if the number of unemployed workers of the same type-country and the number of unmatched entrepreneurs of the same type-country are equal. We incorporate the matching process before bargaining over the total surplus as in Rubinstein and Wolinsky (1985) in which they model matching process as a stochastic process. However, unlike Rubinstein and Wolinsky (1985), we conceptualize the matching process as deterministic such that there is a sufficiently large number of agents of each type-country, and thus agents can frictionlessly search and match with a new partner. Rubinstein and Wolinsky (1985) argue that, when agents are infinitely patient, the bargaining power between a buyer and a seller can be represented as the ratio between the number of buyers and the number of sellers. The bargaining solution in this paper is similarly expressed as the number of unmatched entrepreneurs and the number of unmatched workers. Lastly, Gale (1986a,b, 1987) argue that any perfect equilibrium of the bargaining game implements a Walras allocation of the exchange economy. Similar to his insight, our model's pairwise equilibrium achieves a Pareto efficient frontier.

3 The Model

The model builds upon Galichon, Kominers and Weber (2018)'s framework where they propose a static imperfectly transferable utility model of a one-to-one matching market. We extend their matching framework to allow for multiple countries and multiple sectors and incorporate production function along with a bargaining problem into each one-to-one matching. We derive endogenous sharing rules between two agents explicitly in a pairwise stable equilibrium. From international trade theory perspective, we extend a multi-country, multi-sector, and multi-factor neoclassical trade model of Costinot (2009) to allow for complementarity effect between factors and add a matching with bargaining problem into the model.

3.1 Environment

3.1.1 Agents

There are G countries which are indexed by $g, h \in \mathcal{G} := \{1, 2, ..., G\}$ and S sectors (or goods) which are indexed by $\Sigma^s \in \mathcal{S} := \{1, 2, ..., S\}$ in the world. In each country, there are two types of agents: workers are indexed by I and entrepreneurs are indexed by J.

Workers are heterogeneous and their types are indexed by $x \in \mathcal{X} := \{1, 2, ..., X\}$. In addition, each worker has different preference. Let $I_{xg}(i)$ be an individual worker $i \in \mathcal{I} := \{1, 2, ..., N\}$ of type x in country g where N is the total number of workers in the world.

Entrepreneurs are heterogeneous and their types are indexed by $y \in \mathcal{Y} := \{1, 2, ..., Y\}$. In addition, each entrepreneur has different preference.

Let $J_{yh}(j)$ be an individual entrepreneur $j \in \mathcal{J} := \{1, 2, ..., M\}$ of type y in country h where M is the total number of enterpreneurs in the world.

Let $I_{xg} \in \mathcal{X} \times \mathcal{G}$ denote workers of type x in country g and $J_{yh} \in \mathcal{Y} \times \mathcal{G}$ denote entrepreneurs of type y in country h. In each country g, there are N_{xg} mass of inelastic workers of type x and M_{yg} mass of inelastic entrepreneurs of type y. Total mass of workers in country g is $\sum_{x \in \mathcal{X}} N_{xg} = N_g$ and total mass of entrepreneurs in country g is $\sum_{y \in \mathcal{Y}} M_{yg} = M_g$. We assume that there is a sufficiently large number of agents of each type in each country, denoted as "Large Matching Markets" (See Choo and Siow, 2006; Kojima, Pathak and Roth, 2013; Galichon and Salanié, 2015; Menzel, 2015; Azevedo and Leshno, 2016; Lee, 2016). We further assume that agents' types and countries are publicly observable, so agents can observe the type of their matching counterparts.

3.1.2 Surplus Function

We consider a one-to-one matching between a worker and an entrepreneur. A production function $q(I_{xg}, J_{yh}, \Sigma^s, \tau^s_{gh})$ specifies output that a worker $I_{xg}(i)$ and an entrepreneur $J_{yh}(j)$ can jointly produce when two agents match in sector s where τ^s_{gh} denotes cross-country matching cost between country g and h in sector s. Note that the production function does not depend on individual characteristics, i.e. i or j. We assume that goods can freely move across borders which ensures that the world price of good s is given by $p(\Sigma^s) > 0$. Thus, the surplus function is given by $p(\Sigma^s)q(I_{xg}, J_{yh}, \Sigma^s, \tau^s_{gh})$. After they create joint surplus, they bargain over a division of the total surplus to each side. We denote a transfer from an entrepreneur $J_{yh}(j)$ to a worker $I_{xg}(i)$ as $w^s_{xg(i),yh(j)} \in \mathbb{R}$. If both agents agree upon, then the surplus is frictionless divided between worker $I_{xg}(i)$ and entrepreneur $J_{yh}(j)$. The share for worker, which we denote wage, is $w^s_{xg(i),yh(j)} \in \mathbb{R}$ and the share for entrepreneur, which we denote profit, is $\pi^s_{xg(i),yh(j)} := p(\Sigma^s)q(I_{xg}, J_{yh}, \sigma^s, \tau^s_{gh}) - w^s_{xg(i),yh(j)} \in \mathbb{R}$. We assume a complete contract on contractual agreement between an worker and an entrepreneur. If either side rejects the agreement, then worker $I_{xg}(i)$ and entrepreneur $J_{yh}(j)$ break up and search for new matching partners independently. It is assumed that all agents are infinitely patient. This implies that new search is costless regardless of the number of searches. Agents can search and bargain as long as they like. Frictionless search and complete information implies that there is unique transfer $w^s_{xg,yh} \in \mathbb{R}$ in $S \times X \times G \times Y \times G$ matching markets, which does not depend on individual identity i and j. Suppose that worker $I_{xg}(i)$ and entrepreneur $J_{yh}(j)$ match in sector s and they agree upon transfer $w^s_{xg(i),yh(j)}$ and worker $I_{xg}(k)$ and entrepreneur $J_{yh}(l)$ match in sector s and they agree upon transfer $w^s_{xg(k),yh(l)} > w^s_{xg(i),yh(j)}$. In this case, there is a blocking pair, worker $I_{xg}(i)$ and entrepreneur $J_{yh}(l)$, of agents who would be able to reach a feasible pair that dominates the current matching. For instance, we can find a transfer $w^s_{xg(i),yh(l)}$ with $w^s_{xg(k),yh(l)} > w^s_{xg(i),yh(l)} > w^s_{xg(i),yh(j)}$. Hence, there must be unique transfer $w^s_{xg,yh} \in \mathbb{R}$ in each matching market.

3.1.3 Preferences

Each worker $I_{xg}(i)$ has the following utility function:

$$U_{xg}(i) = \underbrace{\theta \times \ln \left(\sum_s q_{xg}(i, \Sigma^s)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}}_{\text{Systematic Utility}} + \underbrace{\varepsilon_{xg}(i)}_{\text{Idiosyncratic Utility}},$$

where $\theta > 0$ denotes a relative weight on the systematic utility, $\sigma > 1$ denotes the elasticity of substitution, $q_{xg}(i, \Sigma^s)$ is the consumption of good s, and $\varepsilon_{xg}(i)$ represents the idiosyncratic utility.

Consumption for good s is given by,

$$q_{xg}(i, \Sigma^s) = \frac{p(\Sigma^s)^{-\sigma}}{P^{1-\sigma}} E_{xg}(i)$$

where P is the price index with $P^{1-\sigma} := \sum_s p(\Sigma^s)^{1-\sigma}$ and $E_{xg}(i)$ is the total income for worker $I_{xg}(i)$. Plugging the consumption of good s into the above utility function, we can represent the above utility function as indirect utility function (by duality theorem):

$$U_{xg}(i) = \underbrace{\theta \times \ln \frac{E_{xg}(i)}{P}}_{\text{Systematic Utility}} + \underbrace{\varepsilon_{xg}(i)}_{\text{Idiosyncratic Utility}}.$$

Using the same logic, each entrepreneur $J_{yh}(j)$ has the following indirect utility function:

$$V_{yh}(j) = \underbrace{\theta \times \ln \frac{E_{yh}(j)}{P}}_{\text{Systematic Utility}} + \underbrace{\varepsilon_{yh}(j)}_{\text{Idiosyncratic Utility}}.$$

3.1.4 Strategies

Agents maximize the above indirect utility functions by matching with partners. For individual worker $I_{xg}(i)$, there are $MS + 1$ number of strategies: M is the total number of entrepreneurs in the world, and S is the number of sectors. The total number of every combination of possible matching is MS, and the possibility of remaining unmatched is also a strategy. Similarly, for individual entrepreneur $J_{yh}(j)$, there are $NS + 1$ number of strategies. The following two assumptions reduce the dimensions of strategies from an individual-level to a type-country-sector level.

Assumption 1. *Systematic utility part does not depend on* (i, j).

Assumption 2. *Idiosyncratic utility part does not depend on partner's identity* i *or* j.

The first assumption originates from the surplus function and transfer structure which do not depend on individual characteristics. We know that there exists a unique transfer $w^s_{xg,yh} \in \mathbb{R}$ in $S \times X \times G \times Y \times G$ matching markets. Hence, an individual worker $I_{xg}(i)$ receives the same wage when he or she is matched with $J_{yh}(j)$ or $J_{yh}(l)$ in sector s. The second assumption maintains that the unobserved heterogeneity for worker i varies with entrepreneur's type-country-sector, not with entrepreneur's identity j, which implies that a given worker i is indifferent between any entrepreneurs with the same observable type-country-sector.

Assumption 3. *If an agent remains unmatched, he or she receives zero systematic utility, i.e.* $E = P$.

3.1.5 Feasible Bargaing Set

If a worker $I_{xg}(i)$ and an entrepreneur $J_{yh}(j)$ decide to match in sector s, then they bargain over a set of feasible utilities $(\mathcal{U}, \mathcal{V}) \in \mathcal{F}^s_{xg,yh}$ where the feasible bargaining set $\mathcal{F}^s_{xg,yh}$ is defined as follows:

$$\mathcal{F}^s_{xg,yh} := \{(\mathcal{U}, \mathcal{V}) \in \mathbb{R}^2 \mid (\exp(\mathcal{U}))^{\frac{1}{\theta}} + (\exp(\mathcal{V}))^{\frac{1}{\theta}} \leq \frac{p(\Sigma^s)q(I_{xg}, J_{yh}, \Sigma^s, \tau^s_{gh})}{P}\}.$$

The Pareto efficient frontier in this case is $(\exp(\mathcal{U}))^{\frac{1}{\theta}} + (\exp(\mathcal{V}))^{\frac{1}{\theta}} = \frac{p(\Sigma^s)q(I_{xg}, J_{yh}, \Sigma^s, \tau^s_{gh})}{P}$. In Figure 1, we illustrate the feasible bargaining set $\mathcal{F}^s_{xg,yh}$ (shaded region) and the Pareto efficient frontier (blue line) when $\frac{p(\Sigma^s)q(I_{xg}, J_{yh}, \Sigma^s, \tau^s_{gh})}{P} = 100$ and $\theta = 0.5$.

3.1.6 Preference Heterogeneity

Define preference heterogeneity for worker i of type x in country g, $I_{xg}(i)$, as follows:

$$\varepsilon_{xg}(i) = (\varepsilon^0_{xg,00}(i), ..., \varepsilon^s_{xg,yh}(i), ..., \varepsilon^S_{xg,YG}(i)),$$

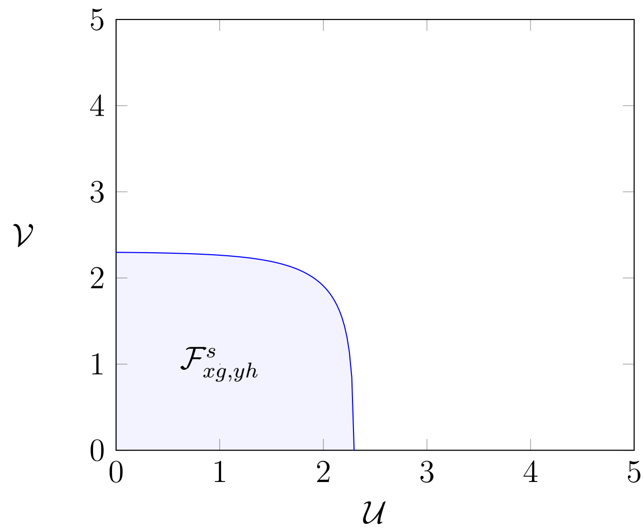
which is a $(SYG + 1) \times 1$ vector.[1] Likewise, we can define preference heterogeneity for entrepreneur j of type y in country h, $J_{yh}(j)$, as follows:

$$\varepsilon_{yh}(j) = (\varepsilon^0_{yh,00}(j), ..., \varepsilon^s_{yh,xg}(j), ..., \varepsilon^S_{yh,XG}(j)),$$

which is a $(SXG + 1) \times 1$ vector. We assume that each component is an independently and identically distributed random variable with a type I

[1] From Assumption 2, the dimension of idiosyncratic part reduces to a type-country-sector.

Figure 1: Feasible Bargaining Set and Pareto Efficient Frontier



Notes: We assume that the total surplus is equal to 100 and the relative weight on the systematic utility parameter θ is set to 0.5.

extreme-value distribution as follows:

$$F(\varepsilon) = \exp\left(-\exp\left(-(\varepsilon+\gamma)\right)\right),$$

where the mean is given by $E(\varepsilon) = 0$ and $\gamma \approx 0.577$, Euler's constant, and the variance is given by $V(\varepsilon) = \dfrac{\pi^2}{6}$ where $\pi \approx 3.14$.[2] The stochastic part ensures that observationally identical workers of type x in country g can match with different type-country entrepreneurs in equilibrium. Furthermore, some agents end up remaining unmatched since the support of the distribution is $(-\infty, \infty)$, implying that all combinations of matches can be observed.

[2]Note that we change the location parameter of standard Gumbel distribution to set the expected value to be zero.

3.1.7 Matching

Let $\mu^s_{xg,yh}$ be the mass of matches between workers of type x in country g and entrepreneurs of type y in country h in sector s. A matching is $\mathcal{M} = \left(\mu^s_{xg,yh}\right)_{s\in\mathcal{S},xg\in\mathcal{X}\times\mathcal{G},yh\in\mathcal{Y}\times\mathcal{G}}$, satisfying the set of $\mu^s_{xg,yh} \geq 0$ such that $\sum_{s\in\mathcal{S}}\sum_{yh\in\mathcal{Y}\times\mathcal{G}}\mu^s_{xg,yh} \leq N_{xg}$ and $\sum_{s\in\mathcal{S}}\sum_{xg\in\mathcal{X}\times\mathcal{G}}\mu^s_{xg,yh} \leq M_{yh}$ for all $xg \in \mathcal{X}\times\mathcal{G}$ and $yh \in \mathcal{Y}\times\mathcal{G}$. We denote the strict interior of $\mathcal{M}$, $\mathcal{M}^0$, as interior matchings satisfying the set of $\mu^s_{xg,yh} > 0$ such that $\sum_{s\in\mathcal{S}}\sum_{yh\in\mathcal{Y}\times\mathcal{G}}\mu^s_{xg,yh} < N_{xg}$ and $\sum_{s\in\mathcal{S}}\sum_{xg\in\mathcal{X}\times\mathcal{G}}\mu^s_{xg,yh} < M_{yh}$ for all $xg \in \mathcal{X}\times\mathcal{G}$ and $yh \in \mathcal{Y}\times\mathcal{G}$.

For any matching $\mathcal{M}$, we denote $U_{xg}(i)$ be the utility level in this matching $\mathcal{M}$ and $U^0_{xg,00}(i)$ be the reservation utility level for worker i of type x in country g. Likewise, for any matching $\mathcal{M}$, we define $V^s_{yh}(j)$ be the utility (or payoff) in this matching $\mathcal{M}$ and $V^0_{00,yh}(j)$ be the reservation utility for entrepreneur j of type y in country h.

Definition 1. *A matching $\mathcal{M} = \left(\mu^s_{xg,yh}\right)_{s\in\mathcal{S},xg\in\mathcal{X}\times\mathcal{G},yh\in\mathcal{Y}\times\mathcal{G}}$ is stable if there exists a transfer vector $\left(w^s_{xg,yh}\right)_{s\in\mathcal{S},xg\in\mathcal{X}\times\mathcal{G},yh\in\mathcal{Y}\times\mathcal{G}}$ and a price vector $(p(\Sigma^s))_{s\in\mathcal{S}}$ such that*

i) Individual Rationality : For all worker i and entrepreneur j who are matched, we have $U_{xg}(i) \geq U^0_{xg,00}(i)$ and $V_{yh}(j) \geq V^0_{00,yh}(j)$,

ii) Pairwise Stability : There is no blocking coalition (i,j) of workers and entrepreneurs who would be able to reach a feasible pair of utilities dominating $U_{xg}(i)$ and $V_{yh}(j)$.

Following Choo and Siow (2006)'s original insight, finding a stable matching $\mathcal{M}$ is equivalent to solving discrete choice problem on both sides of the market. Hence, we follow the same steps as in Choo and Siow (2006) to characterize a stable matching $\mathcal{M}$.

3.2 Utility Maximization (Worker)

Consider utility maximization problem for worker i of type x in country g. Let his utility when he matches with a type y entrepreneur in country h in sector s be

$$U^s_{xg,yh}(i) = \theta \times \ln\frac{w^s_{xg,yh}}{P} + \varepsilon^s_{xg,yh}(i), \quad \theta > 0,$$

where θ measures the relative weight on systematic utility, $w^s_{xg,yh}$ is the equilibrium transfer (wage) that a type y entrepreneur in country h must pay to match with a type x worker in country g in sector s, and $\varepsilon^s_{xg,yh}(i)$ is preference heterogeneity in worker i of type x in country g's preference over type y entrepreneur in country h in sector s. The utility for remaining unmatched is specified as follows:

$$U^0_{xg,00}(i) = \theta \times \ln \frac{w^0_{xg,00}}{P} + \varepsilon^0_{xg,00}(i) = \varepsilon^0_{xg,00}(i), \qquad w^0_{xg,00} = P,$$

where $w^0_{xg,00}$ is the reservation wage level if the worker remains unmatched and $\varepsilon^0_{xg,00}(i)$ is idiosyncratic utility when the worker chooses to remain unmatched.

Worker i of type x in country g, $I_{xg}(i)$, will maximize his (or her) utility among $(SYG + 1)$ number of available matching alternatives:

$$U_{xg}(i) = \max_{\substack{s\in\mathcal{S} \\ yh\in\mathcal{Y}\times\mathcal{G}}} \{U^0_{xg,00}(i), ..., U^s_{xg,yh}(i), ..., U^S_{xg,YG}(i)\}.$$

Assume that there is a sufficiently large number of entrepreneurs of each type in each country. Let $\Pr[U_{xg}(i) = U^s_{xg,yh}(i)]$ be the probability of choosing a type y entrepreneur in country h in sector s and $\Pr[U_{xg}(i) = U^0_{xg,00}(i)]$ be the probability of remaining unmatched by worker i of type x in country g. Following McFadden (1974), we can derive conditional choice probabilities as follows (See Appendix 6.1 for detailed derivation):

$$\Pr[U_{xg}(i) = U^s_{xg,yh}(i)] = \frac{\left(w^s_{xg,yh}\right)^\theta}{P^\theta + \sum_{(t,z,k)\in\mathcal{S}\times\mathcal{Y}\times\mathcal{G}\setminus(0,0,0)} \left(w^t_{xg,zk}\right)^\theta},$$

$$\Pr[U_{xg}(i) = U^0_{xg,00}(i)] = \frac{P^\theta}{P^\theta + \sum_{(t,z,k)\in\mathcal{S}\times\mathcal{Y}\times\mathcal{G}\setminus(0,0,0)} \left(w^t_{xg,zk}\right)^\theta},$$

Let $(\mu^s_{xg,yh})_{supply} := \Pr[U_{xg}(i) = U^s_{xg,yh}(i)] \times N_{xg}$ be the mass of type x in country g workers who would like to supply for type y entrepreneurs in country h in sector s. Similarly, let $(\mu^0_{xg,00})_{supply} := \Pr[U_{xg}(i) = U^0_{xg,00}(i)] \times$

N_{xg} be the mass of type x in country g workers who want to remain unmatched. Then, supply equation by type x in country g workers for type y entrepreneurs in country h in sector s is given by,

$$(\mu^s_{xg,yh})_{supply} = (\mu^0_{xg,00})_{supply} \times \left(\frac{w^s_{xg,yh}}{P}\right)^\theta. \quad (1)$$

By taking the log of both sides of the equation,

$$\ln \frac{(\mu^s_{xg,yh})_{supply}}{(\mu^0_{xg,00})_{supply}} = \theta \times \ln\left(\frac{w^s_{xg,yh}}{P}\right).$$

The parameter θ captures the extensive margin elasiticity of labor supply with respect to the real wage.

3.3 Utility Maximization (Entrepreneur)

Next, consider utility maximization problem for entrepreneur j of type y in country h. Let his utility when he matches with a type x worker in country g in sector s be

$$V^s_{xg,yh}(j) = \theta \times \ln \frac{\pi^s_{xg,yh}}{P} + \varepsilon^s_{xg,yh}(j), \quad \theta > 0,$$

where θ measures the relative weight on systematic utility, $\pi^s_{xg,yh} := p(\Sigma^s)q(I_{xg}, J_{yh}, \Sigma^s, \tau^s_{gh}) - w^s_{xg,yh}$ is the equilibrium profit that goes to entrepreneur, and $\varepsilon^s_{xg,yh}(j)$ is preference heterogeneity in entrepreneur j of type y in country h's preference over type x worker in country g in sector s. The utility for remaining unmatched is specified as follows:

$$V^0_{00,yh}(j) = \theta \times \ln \frac{\pi^0_{00,yh}}{P} + \varepsilon^0_{00,yh}(j) = \varepsilon^0_{00,yh}(j), \quad \pi^0_{00,yh} = P,$$

where $\pi^0_{00,yh}$ is the reservation profit level if the entrepreneur remains unmatched and $\varepsilon^0_{00,yh}(j)$ is idiosyncratic utility when the entrepreneur chooses to remain unmatched. Entrepreneur j of type y in country h, $J_{yh}(j)$, will maximize his (or her) utility among $(SXG+1)$ number of available matching alternatives:

$$V_{yh}(j) = \max_{\substack{s\in\mathcal{S} \\ xg\in\mathcal{X}\times\mathcal{G}}} \{V^0_{00,yh}(j), ..., V^s_{xg,yh}(j), ..., V^S_{XG,yh}(j)\}.$$

Using the same step in worker utility maximization problem, we can derive conditional choice probabilities for entrepreneurs. Let $\Pr[V_{yh}(j) = V^s_{xg,yh}(j)]$ be the probability of choosing a type x worker in country g in sector s and $\Pr[V_{yh}(j) = V^0_{00,yh}(j)]$ be the probability of remaining unmatched by any entrepreneur j of type y in country h:

$$\Pr[V_{yh}(j) = V^s_{xg,yh}(j)] = \frac{\left(\pi^s_{xg,yh}\right)^\theta}{P^\theta + \sum_{(t,z,k)\in\mathcal{S}\times\mathcal{X}\times\mathcal{G}\setminus(0,0,0)} \left(\pi^t_{zk,yh}\right)^\theta},$$

$$\Pr[V_{yh}(j) = V^0_{00,yh}(j)] = \frac{P^\theta}{P^\theta + \sum_{(t,z,k)\in\mathcal{S}\times\mathcal{X}\times\mathcal{G}\setminus(0,0,0)} \left(\pi^t_{zk,yh}\right)^\theta}.$$

Let $(\mu^s_{xg,yh})_{demand} := \Pr[V_{yh}(j) = V^s_{xg,yh}(j)] \times M_{yh}$ be the mass of type y in country h entrepreneurs who are willing to demand for type x worker in country g in sector s and $(\mu^0_{00,yh})_{demand} := \Pr[V_{yh}(j) = V^0_{00,yh}(j)] \times M_{yh}$ be the mass of type y in country h entrepreneurs who want to remain unmatched. Then, demand equation by type y entrepreneurs in country h for type x in country g workers in sector s is given by,

$$(\mu^s_{xg,yh})_{demand} = (\mu^0_{00,yh})_{demand} \times \left(\frac{\pi^s_{xg,yh}}{P}\right)^\theta. \tag{2}$$

3.4 Labor Market Clearing

There are $S\times X\times G\times Y\times G$ matching markets for every combination of types of workers and entrepreneurs. Given equilibrium wages $\left(w^s_{xg,yh}\right)_{s\in\mathcal{S},xg\in\mathcal{X}\times\mathcal{G},yh\in\mathcal{Y}\times\mathcal{G}}$ and equilibrium prices $(p(\Sigma^s))_{s\in\mathcal{S}}$, $S\times X\times G\times Y\times G$ matching markets clear when supply by type x in country g workers for type y entrepreneurs in country h is equal to demand by type y entrepreneurs in country h for type

x in country g workers for all matching markets, i.e. $\mu^s_{xg,yh} = (\mu^s_{xg,yh})_{supply} = (\mu^s_{xg,yh})_{demand}$ for all s, x, g, y, and h. Using equations (1) and (2), we can derive the following matching function - an equilibrium relationship between the mass of matched $\mu^s_{xg,yh}$ and the mass of unmatched workers and entrepreneurs ($\mu^0_{xg,00}$ and $\mu^0_{00,yh}$).

$$\mu^s_{xg,yh} = \mu^s_{xg,yh}(\mu^0_{xg,00}, \mu^0_{00,yh}) = \left(\frac{p(\Sigma^s)q(I_{xg}, J_{yh}, \Sigma^s, \tau^s_{gh})}{P}\right)^{\theta} \times \left[(\mu^0_{xg,00})^{-\frac{1}{\theta}} + (\mu^0_{00,yh})^{-\frac{1}{\theta}}\right]^{-\theta}, \tag{3}$$

for all s, x, g, y, and h. First, note that the matching function $\mu^s_{xg,yh}(\mu^0_{xg,00}, \mu^0_{00,yh})$ satisfies homogeneity of degree one in the mass of unmatched workers and the mass of unmatched entrepreneurs (constant returns to scale), which implies that there is no scale effect in the mass of agents. Second, the equilibrium matching $\mathcal{M} = \left(\mu^s_{xg,yh}\right)_{s\in\mathcal{S}, xg\in\mathcal{X}\times\mathcal{G}, yh\in\mathcal{Y}\times\mathcal{G}}$ is stable since all agents maximize their utilities, and thus satisfying both the *individual rationality* condition and the *pairwise stability* condition. Third, the matching function provides a micro foundation for Mortensen and Pissarides (1994)-type matching function in which the number of matches is the function of unemployment and vacancy. Unlike an exogenous Mortensen and Pissarides (1994)-type matching function, our matching function is derived from the supply and demand framework.

Given the equilibrium matching function, we can characterize equilibrium wages $\left(w^s_{xg,yh}\right)$ and profits $\left(\pi^s_{xg,yh}\right)$ for all $S \times X \times G \times Y \times G$ matching markets. By plugging the equilibrium matching function in equation (3) into the supply equation and the demand equation in (1) and (2), respectively, we can derive the following equilibrium wages and profits:

$$w^s_{xg,yh} = \underbrace{p(\Sigma^s)q(I_{xg}, J_{yh}, \Sigma^s, \tau^s_{gh})}_{\text{Total surplus}} \underbrace{\frac{(\mu^0_{xg,00})^{-\frac{1}{\theta}}}{(\mu^0_{xg,00})^{-\frac{1}{\theta}} + (\mu^0_{00,yh})^{-\frac{1}{\theta}}}}_{\text{Bargaining power}},$$

$$\pi^s_{xg,yh} = \underbrace{p(\Sigma^s)q(I_{xg}, J_{yh}, \Sigma^s, \tau^s_{gh})}_{\text{Total surplus}} \underbrace{\frac{(\mu^0_{00,yh})^{-\frac{1}{\theta}}}{(\mu^0_{xg,00})^{-\frac{1}{\theta}} + (\mu^0_{00,yh})^{-\frac{1}{\theta}}}}_{\text{Bargaining power}}, \tag{4}$$

for all s, x, g, y, and h. The bargaining power between a type x in country g worker and a type y entrepreneurs in country h in sector s is endogenously determined by the mass of unmatched workers of the same type-country $\mu^0_{xg,00}$ (supply) and the mass of unmatched entrepreneurs of the same type-country $\mu^0_{00,yh}$ (demand). The endogenous bargaining power can be interpreted as a supply and demand framework. Suppose that there are more unmatched workers of type x in country g (supply) than unmatched entrepreneurs of type y entrepreneurs in country h (demand) $\mu^0_{xg,00} > \mu^0_{00,yh}$ in a matching market where type x in country g workers match with type y in country h entrepreneurs in sector s. In each two-person bargaining problem, this implies that there are more outside alternatives, $\mu^0_{xg,00}$, for entrepreneurs, which increases the bargaining power of entrepreneurs. From the worker's perspective, there are less outside alternatives, $\mu^0_{00,yh}$, which decreases the bargaining power of workers.

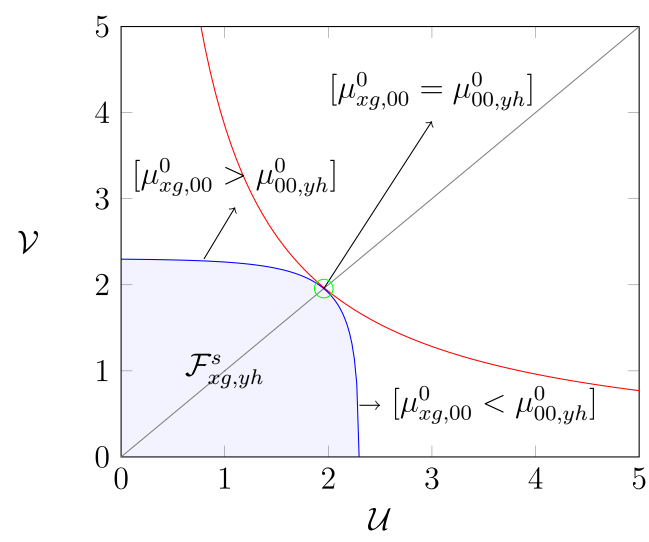
The solution of the classic Nash two-person bargaining problem is symmetric in Nash (1950). In addition, if two agents are infinitely patient as in our model, Rubinstein (1982)'s strategic bargaining solution is identical to Nash (1950)'s solution as follows:

$$w^s_{xg,yh} = \frac{1}{2}p(\Sigma^s)q(I_{xg}, J_{yh}, \Sigma^s, \tau^s_{gh}) \quad \text{and} \quad \pi^s_{xg,yh} = \frac{1}{2}p(\Sigma^s)q(I_{xg}, J_{yh}, \Sigma^s, \tau^s_{gh}).$$

The equilibrium sharing rule is equivalent to the above symmetric solution *only if* the mass of unmatched workers of the same type-country $\mu^0_{xg,00}$ is equal to the mass of unmatched entrepreneurs of the same type-country

$\mu^0_{00,yh}$ (See Figure 2). The structure of the matching and bargaining problem can explain the discrepancy between our equilibrium sharing rule and the symmetric two-person bargaining solution. Because our model's bargaining problem is nested in a matching market and each matching market is also affected by other matching markets, the specific two-person bargaining solution is determined by a system-wide network structure. Rubinstein and Wolinsky (1985) find that, in a random matching with a sequential bargaining process, bargaining power is determined by the relative size of buyers and sellers as players become infinitely patient. Manea (2011) extends the idea of Rubinstein and Wolinsky (1985) to a network structure and demonstrate that the shortage ratio of the mutually estranged set, defined as the ratio of the number of partners to estranged players, determines the collective bargaining power of its members. Similarly, our model's bargaining solution depends on the ratio of the number of unmatched workers to the number of unmatched entrepreneurs.

Figure 2: The Equilibrium Sharing Rule



Notes: We assume that the total surplus is equal to 100 and the relative weight on systematic utility parameter θ is set to 0.5. The green dot, $(\ln(50^{0.5}), \ln(50^{0.5}))$, indicates the symmetric two-person bargaining solution. The equilibrium sharing rule can have any points along the Nash equilibria and is endogenously determined via $\mu^0_{xg,00}$ and $\mu^0_{00,yh}$.

3.5 Goods Market Clearing

Since goods can freely move across countries and agents have the same systematic CES preferences, the total value of demand for good s is as follows:

$$p(\Sigma^s)q(\Sigma^s) = \frac{p(\Sigma^s)^{1-\sigma}}{P^{1-\sigma}}E$$

where $E := P\left(\sum_x \sum_g \mu^0_{xg,00} + \sum_y \sum_h \mu^0_{00,yh}\right)$ $+\sum_s \sum_x \sum_g \sum_y \sum_h \mu^s_{xg,yh}(\mu^0_{xg,00}, \mu^0_{00,yh})p(\Sigma^s)q(I_{xg}, J_{yh}, \Sigma^s, \tau^s_{gh})$ is the world total income. The total value of supply for good s is as follows:

$$\sum_x \sum_g \sum_y \sum_h \mu^s_{xg,yh}(\mu^0_{xg,00}, \mu^0_{00,yh})p(\Sigma^s)q(I_{xg}, J_{yh}, \Sigma^s, \tau^s_{gh}).$$

S goods markets clear when for all $s \in \mathcal{S}$,

$$\frac{p(\Sigma^s)^{1-\sigma}}{P^{1-\sigma}}E = \sum_x \sum_g \sum_y \sum_h \mu^s_{xg,yh}(\mu^0_{xg,00}, \mu^0_{00,yh})p(\Sigma^s)q(I_{xg}, J_{yh}, \Sigma^s, \tau^s_{gh}). \quad (5)$$

3.6 Equilibrium

Definition 2. *The matching function equilibrium is a solution of the following $XG + YG + S$ system of nonlinear equations with XG number of unknowns $\mu^0_{xg,00}$, YG number of unknowns $\mu^0_{00,yh}$, and S number of unknonws $p(\Sigma^s)$.*

$$\begin{cases} \mu^0_{xg,00} + \sum_s \sum_y \sum_h \mu^s_{xg,yh}(\mu^0_{xg,00}, \mu^0_{00,yh}) = N_{xg}, & \forall x \in \mathcal{X} \text{ and } \forall g \in \mathcal{G}, \\ \mu^0_{00,yh} + \sum_s \sum_x \sum_g \mu^s_{xg,yh}(\mu^0_{xg,00}, \mu^0_{00,yh}) = M_{yh}, & \forall y \in \mathcal{Y} \text{ and } \forall h \in \mathcal{G}, \\ \frac{p(\Sigma^s)^{1-\sigma}}{P^{1-\sigma}}E = \sum_x \sum_g \sum_y \sum_h \mu^s_{xg,yh}(\mu^0_{xg,00}, \mu^0_{00,yh})p(\Sigma^s)q(I_{xg}, J_{yh}, \Sigma^s, \tau^s_{gh}), & \forall s \in \mathcal{S}. \end{cases}$$

The equilibrium outcome of the two-sided matching with bargaining game in our model can also be interpreted as a Walrasian equilibrium. Given equilibrium wages $\left(w^s_{xg,yh}\right)$ and prices $(p(\Sigma^s))$, all agents maximize utilities and all markets clear. Gale (1986a,b, 1987) studied a model of random matching and bargaining when the number of agents is large, and he showed that, under certain conditions (e.g., agents do not discount future), any perfect equilibrium of the bargaining game implements a Walras allocation of the exchange economy.

3.7 Welfare in Equilibrium

In equilibrium, the (ex-ante) expected utility for worker i of type x in country g before she observes her realizations of $\varepsilon_{xg}(i)$ can be expressed as (See Appendix 6.2 for detailed derivation):

$$\begin{aligned}\mathbb{E}\left[U_{xg}(i)\right] &= \mathbb{E}\left[\max_{\substack{s\in\mathcal{S}\\ yh\in\mathcal{Y}\times\mathcal{G}}}\left\{U^0_{xg,00}(i),...,U^s_{xg,yh}(i),...,U^S_{xg,YG}(i)\right\}\right]\\ &= \ln\left[\sum_{(s,y,h)\in\mathcal{S}\times\mathcal{Y}\times\mathcal{G}\cup\{(0,0,0)\}}\left(\frac{w^s_{xg,yh}}{P}\right)^{\theta}\right] = \ln\frac{N_{xg}}{\mu^0_{xg,00}}.\end{aligned}$$

Similarly, the (ex-ante) expected utility for entrepreneur j for type y in country h before she observes her realizations of $\varepsilon_{yh}(j)$ is as follows:

$$\begin{aligned}\mathbb{E}\left[V_{yh}(j)\right] &= \mathbb{E}\left[\max_{\substack{s\in\mathcal{S}\\ xg\in\mathcal{X}\times\mathcal{G}}}\left\{V^0_{00,yh}(j),...,V^s_{xg,yh}(j),...,V^S_{XG,yh}(j)\right\}\right]\\ &= \ln\left[\sum_{(s,x,g)\in\mathcal{S}\times\mathcal{X}\times\mathcal{G}\cup\{(0,0,0)\}}\left(\frac{\pi^s_{xg,yh}}{P}\right)^{\theta}\right] = \ln\frac{M_{yh}}{\mu^0_{00,yh}}.\end{aligned}$$

The parameter θ corresponds to the elasticity of substitution parameter in the CES-type expected utility function, and it measures how agents evaluate the ex-post distribution of income before he or she observes her realizations of idiosyncratic preference. Suppose that the parameter θ is small - agents' utilities depend more on the idiosyncratic preference than income. Then, agents prefer the ex-post equal distribution of income. Behind a veil of ignorance, i.e., no one knows his matching partners and sectors ex-ante, if randomness mostly determines utilities, agents would prefer equal income distribution.

It is worth noting that the logsum formula derived here is identical to that of Small and Rosen (1981) in which they study the measurement of welfare changes in a discrete choice model. Following Small and Rosen (1981), we can use changes in (ex-ante) expected utilities $\mathbb{E}[U_{xg}],\ \forall(x,g) \in \mathcal{X}\times\mathcal{G}$ and $\mathbb{E}[V_{yh}],\ \forall(y,h) \in \mathcal{Y}\times\mathcal{G}$ in response to changes in economic system as welfare metrics for all agents.

At the country-level, a total welfare metric in country g can be defined as follows:

$$\mathcal{W}_g := \sum_{x\in\mathcal{X}} N_{xg}\,\mathbb{E}[U_{xg}] + \sum_{y\in\mathcal{Y}} M_{yg}\,\mathbb{E}[V_{yg}].$$

The welfare metric in country g is "inequality-adjusted" as in Galle, Rodriguez-Clare and Yi (2017), Jones and Klenow (2016), and Antras, De Gortari and Itskhoki (2017). To understand the concept, suppose that a social planner in country g tries to maximize social welfare. As in Rawls (1971), this social planner who is behind the veil of ignorance rank for every possible combination of equilibrium income distribution. Suppose that two hypothetical equilibria with the same level of aggregate real income with a different variance. Then, the social planner, who follows the above welfare metric with the low parameter θ, would prefer one with less variance. The parameter θ governs how the social planner evaluates inequality. The welfare metric can also be aggregated up to the world-level:

$$\mathcal{W} := \sum_{g\in\mathcal{G}} \mathcal{W}_g.$$

4 Comparative Statics

We illustrate two simple comparative statics results that our model can be applied in the international economics literature. The first case is the reduction in offshoring cost in specific sector s. The second case is the economic integration agreement such as FTA between two countries. To illustrate the core insights from the model, we consider the following two symmetry assumptions which allow us to obtain analytical solutions.

Assumption 4. *Suppose that q satisfies*

$$q(I_{xg}, J_{yh}, \Sigma^s, \tau^s_{gh}) := \frac{q}{\tau^s_{gh}}, \quad \forall x \in \mathcal{X}, \ \forall g \in \mathcal{G}, \ \forall y \in \mathcal{Y}, \ \forall h \in \mathcal{G} \text{ and } \forall s \in \mathcal{S}.$$

with $\tau^s_{gh} = \tau > 1 \, \forall g \neq h$ and $\tau^s_{gh} = 1 \, \forall g = h$.

Assumption 5. *Suppose that*

$$N_{xg} = M_{yh} = L, \quad \forall x \in \mathcal{X}, \ \forall g \in \mathcal{G}, \ \forall y \in \mathcal{Y} \text{ and } \forall h \in \mathcal{G}.$$

4.1 Reduction in Sector-Specific Offshoring Costs

Proposition 1. *Suppose that Assumptions 4 and 5 hold. If cross-country matching costs in sector 1 drop from $\tau^1_{gh} = \tau$ to $(\tau^1_{gh})' < \tau \, \forall g \neq h$, then*

(i) The relative price of good 1, $\frac{p(\Sigma^1)}{p(\Sigma^s)}$, decreases for any $s \neq 1$;

(ii) The number of unmatched agents reduces for all types of workers and entrepreneurs;

(iii) In sector 1, the number of within-country matching reduces and wages (salaries) for within-country matching decrease while the number of cross-country matching increases and wages (salaries) for cross-country matching rise;

(iv) In other sectors, wages (salaries) for cross-country matching rise. The number of cross-country matching depends on the magnitude of a drop in the matching cost and the parameter θ.

Proof. See Appendix 6.3. □

The ease of offshoring in sector 1 serves as technology advancement in sector 1. The boost in productivity induces more agents to engage in sector 1 in cross-country matching markets. This effect in sector 1 spills over to other sectors. The equilibrium terms of trade changes and thus the relative price of good 1 decreases. In other sectors, due to the relative price changes, agents are more likely to match. The productivity effect and the relative price effect reduce the number of unmatched agents for all types of workers and entrepreneurs. Therefore, all agents are better off from the reduction in sector-specific offshoring cost. In the within-country matching market in sector 1, wages (salaries) decrease because only the relative price effect is operating. The reductions in wages (salaries) do not necessarily lead to reductions in welfare because agents have better options in the cross-country matching markets in sector 1 and the matching markets in other sectors.

The distributional result of falling offshoring costs studied in this paper relates to the task-based offshoring model of Grossman and Rossi-Hansberg (2008). They identify the productivity effect, the relative price effect, and the labor supply effect of offshoring in the source country. We also derive the three effects; however, ours differ in several dimensions. They focus on the source country while our model can accommodate an arbitrary number of countries. Their treatment of falling costs of offshoring lies in the task while we focus on the reduction in sector-specific offshoring cost. Most importantly, we derive the result from bargaining and matching with the complementary effect between tasks while their model is limited to allow for the potential patterns of complementarity between tasks. Lastly, even though we incorporate the complex bargaining and matching problem, our model features general equilibrium structure.

4.2 The Third-Country Effects of Economic Integration Agreement

Proposition 2. *Suppose that there are three countries in the world and Assumptions 4 and 5 hold. If cross-country matching costs between country 1 and country 2 drop from $\tau_{12}^s = \tau_{21}^s = \tau$ to $(\tau_{12}^s)' = (\tau_{21}^s)' < \tau \ \forall s$, then*

(i) In countries 1 and 2, the number of unmatched agents decreases for all types of workers and entrepreneurs. In country 3, the number of unmatched agents increases for all types of workers and entrepreneurs;

(ii) The number of cross-country matches between country 1 and country 2 and

the number of within-country matches in country 3 increase. All other matches decrease;

(iii) In within-country matches, wages (salaries) do not change. In cross-country matches between country 1 and country 2, wages (salaries) increase.

(iv) In cross-country matches between country 1 and country 3, wages (salaries) increase for agents in country 1 while wages (salaries) decrease for agents in country 3.

(v) In cross-country matches between country 2 and country 3, wages (salaries) increase for agents in country 2 while wages (salaries) decrease for agents in country 3.

Proof. See Appendix 6.3. □

The reductions in bilateral matching costs between country 1 and country 2 can have effects on agents in country 3 through bargaining powers. The productivity increase in cross-country matching between country 1 and 2 induces more agents in country 1 and 2 to match in these markets from other markets. This effect also reduces the number of unmatched agents in country 1 and 2. Hence, the agents in country 1 and country 2 are better off. The reductions in the number of unmatched agents in country 1 and 2 spill over to country 3. In cross-country matching markets between country 1 (or 2) and 3, the reduction in bargaining power for agents in country 3 reduces wages (salaries). Agents in country 3 revert to country 3 or become unemployed. Hence, agents in country 3 are worse off.

We identify one notable result from this exercise. The un-related third country's domestic production, i.e., within-country matching in country 3, increases from the economic integration agreement between country 1 and country 2. This effect is magnified if the country 3 is more exposed to globalization. The more the number of cross-country matching between the country 3 and other countries is, the higher the impact of the bilateral economic integration agreement on the country 3 is. This example illustrates why our model is useful in analyzing complex problems in international economics using the multi-country framework.

5 Conclusion

This paper develops a multi-country, multi-sector, and multi-factor with a two-sided one-to-one matching model to analyze the welfare impacts of globalization. Through the lens of bargaing and matching framework, our model can address questions in internationa economics literature. We derive two simple comparative statics predictions that offshoring can increase welfare for all agents without conflict of interests and bilateral economic integration agreement can have effects on third-country agents.

6 Appendix

6.1 Derivation of Conditional Choice Probability

$$\Pr\{U_{xg}(i) = U^s_{xg,yh}(i)\} = \mathbb{E}\left[\prod_{\forall(t,z,k)\neq(s,y,h)} F(\varepsilon^s_{xg,yh}(i) + \theta\ln\frac{w^s_{xg,yh}}{P} - \theta\ln\frac{w^t_{xg,zk}}{P})\right]$$
$$= \int_{-\infty}^{\infty}\exp\left[-\sum_{\forall(t,z,k)\neq(s,y,h)}\exp\left(-(\varepsilon+\gamma) - \theta\ln\frac{w^s_{xg,yh}}{P} + \theta\ln\frac{w^t_{xg,zk}}{P}\right)\right]$$
$$\times\exp\left[-(\varepsilon+\gamma) - \exp\left(-(\varepsilon+\gamma)\right)\right]d\varepsilon$$

Let $\xi = 1 + \sum_{\forall(t,z,k)\neq(s,y,h)}\exp\left[-\theta\ln\frac{w^s_{xg,yh}}{P} + \theta\ln\frac{w^t_{xg,zk}}{P}\right]$. Then, $\Pr\{U_{xg}(i) = U^s_{xg,yh}(i)\}$ can be represented as follows:

$$\begin{aligned}\Pr\{U_{xg}(i) = U^s_{xg,yh}(i)\} &= \int_{-\infty}^{\infty}\exp\left[-(\varepsilon+\gamma) - \xi\exp\left(-(\varepsilon+\gamma)\right)\right]d\varepsilon \\ &= \left[\frac{\exp\left(-\xi\exp\left(-(\varepsilon+\gamma)\right)\right)}{\xi}\right]_{-\infty}^{\infty} \\ &= \frac{1}{\xi} \\ &= \frac{\left(\frac{w^s_{xg,yh}}{P}\right)^\theta}{\sum_{\forall(t,z,k)}\left(\frac{w^t_{xg,zk}}{P}\right)^\theta}\end{aligned}$$

6.2 Derivation of Ex-Ante Expected Utility

$$\begin{aligned}
\mathbb{E}\left[U_{xg}(i)\right] &= \mathbb{E}\left[\max_{\substack{s\in\mathcal{S}\\ yh\in\mathcal{Y}\times\mathcal{G}}}\{U^0_{xg,00}(i),...,U^s_{xg,yh}(i),...,U^S_{xg,YG}(i)\}\right]\\
&= \mathbb{E}\left[U^s_{xg,yh}(i)|(t,z,k)=\underset{(t,z,k)\in\mathcal{S}\times\mathcal{Y}\times\mathcal{G}\cup\{(0,0,0)\}}{\operatorname{argmax}} U^t_{xg,zk}(i)\right]\\
&= \theta\ln\frac{w^s_{xg,yh}}{P}+\mathbb{E}\left[\varepsilon^s_{xg,yh}(i)|(t,z,k)=\underset{(t,z,k)\in\mathcal{S}\times\mathcal{Y}\times\mathcal{G}\cup\{(0,0,0)\}}{\operatorname{argmax}} U^t_{xg,zk}(i)\right]\\
&= \theta\ln\frac{w^s_{xg,yh}}{P}+\left(\Pr\{U_{xg}(i)=U^s_{xg,yh}(i)\}\right)^{-1}\int_{-\infty}^{\infty}\varepsilon\exp\left[-(\varepsilon+\gamma)-\xi\exp\left(-(\varepsilon+\gamma)\right)\right]d\varepsilon\\
&= \theta\ln\frac{w^s_{xg,yh}}{P}+\xi\int_{-\infty}^{\infty}\varepsilon\exp\left[-(\varepsilon+\gamma)-\xi\exp\left(-(\varepsilon+\gamma)\right)\right]d\varepsilon\\
&= \theta\ln\frac{w^s_{xg,yh}}{P}-\gamma-\xi\int_{-\infty}^{\infty}-(\varepsilon+\gamma)\exp\left[-(\varepsilon+\gamma)-\xi\exp\left(-(\varepsilon+\gamma)\right)\right]d\varepsilon
\end{aligned}$$

Let $\Delta=-(\varepsilon+\gamma)$. Then, $\mathbb{E}[U_{xg}(i)]$ can be represented as follows:

$$\begin{aligned}
\mathbb{E}[U_{xg}(i)] &= \theta\ln\frac{w^s_{xg,yh}}{P}-\gamma-\xi\int_{-\infty}^{\infty}-(\varepsilon+\gamma)\exp\left[-(\varepsilon+\gamma)-\xi\exp\left(-(\varepsilon+\gamma)\right)\right]d\varepsilon\\
&= \theta\ln\frac{w^s_{xg,yh}}{P}-\gamma-\xi\int_{-\infty}^{\infty}\Delta\exp\left[\Delta-\xi\exp\left(\Delta\right)\right]d\Delta.
\end{aligned}$$

Using $\int_{-\infty}^{\infty}\Delta\exp\left[\Delta-\xi\exp\left(\Delta\right)\right]d\Delta=-\dfrac{\gamma+\ln\xi}{\xi}$,

$$\mathbb{E}[U_{xg}(i)]=\theta\ln\frac{w^s_{xg,yh}}{P}+\ln\xi=\ln\left[\sum_{(s,y,h)\in\mathcal{S}\times\mathcal{Y}\times\mathcal{G}\cup\{(0,0,0)\}}\left(\frac{w^s_{xg,yh}}{P}\right)^{\theta}\right].$$

Using supply equation (1),

$$\mathbb{E}[U_{xg}(i)]=\ln\left[\sum_{(s,y,h)\in\mathcal{S}\times\mathcal{Y}\times\mathcal{G}\cup\{(0,0,0)\}}\frac{\mu^s_{xg,yh}}{\mu^0_{xg,00}}\right]=\ln\frac{N_{xg}}{\mu^0_{xg,00}}.$$

6.3 Proofs

6.3.1 Proof of Proposition 1

Proof. (i) The demand and supply of sector s goods can be expressed as follows:

$$\frac{p(\Sigma^s)^{-\sigma}}{P^{1-\sigma}}E = \underbrace{\sum_x \sum_g \sum_y \sum_h \mu^s_{xg,yh}(\mu^0_{xg,00}, \mu^0_{00,yh}) q(I_{xg}, J_{yh}, \Sigma^s, \tau^s_{gh})}_{\text{World total output in sector 1}}$$

$$= \sum_x \sum_g \sum_y \sum_h \left(\frac{p(\Sigma^s) \frac{q}{\tau^s_{gh}}}{P} \right)^\theta \times \frac{\mu^0_{00,00}}{2^\theta} \times \frac{q}{\tau^s_{gh}}$$

$$= \sum_x \sum_g \sum_y \sum_{h \neq g} \left(\frac{p(\Sigma^s) \frac{q}{\tau^s_{gh}}}{P} \right)^\theta \times \frac{\mu^0_{00,00}}{2^\theta} \times \frac{q}{\tau^s_{gh}} + \sum_x \sum_g \sum_y \sum_{h=g} \left(\frac{p(\Sigma^s) q}{P} \right)^\theta \times \frac{\mu^0_{00,00}}{2^\theta} \times q.$$

Hence, the relative price of good 1 can be represented as:

$$\left(\frac{p(\Sigma^1)}{p(\Sigma^s)} \right)^{-\sigma-\theta} = \frac{\sum_x \sum_g \sum_y \sum_{h \neq g} \left(\frac{\frac{q}{\tau^1_{gh}}}{P} \right)^\theta \times \frac{\mu^0_{00,00}}{2^\theta} \times \frac{q}{\tau^1_{gh}} + \sum_x \sum_g \sum_y \sum_{h=g} \left(\frac{q}{P} \right)^\theta \times \frac{\mu^0_{00,00}}{2^\theta} \times q}{\sum_x \sum_g \sum_y \sum_{h \neq g} \left(\frac{\frac{q}{\tau^s_{gh}}}{P} \right)^\theta \times \frac{\mu^0_{00,00}}{2^\theta} \times \frac{q}{\tau^s_{gh}} + \sum_x \sum_g \sum_y \sum_{h=g} \left(\frac{q}{P} \right)^\theta \times \frac{\mu^0_{00,00}}{2^\theta} \times q}.$$

Because cross-country matching costs in sector 1 drop from $\tau^1_{gh} = \tau$ to $(\tau^1_{gh})' < \tau \ \forall g \neq h$, the relative price of good 1, $\frac{p(\Sigma^1)}{p(\Sigma^s)}$, should decrease.

(ii) The labor market clearing condition can be expressed as:

$$\mu^0_{00,00} + \sum_y \sum_h \mu^1_{xg,yh}(\mu^0_{00,00}) + \sum_{s \neq 1} \sum_y \sum_h \mu^s_{xg,yh}(\mu^0_{00,00}) = L$$

$$\iff \mu^0_{00,00}+\sum_y\sum_h\left(\frac{p(\Sigma^1)\frac{q}{\tau^1_{gh}}}{P}\right)^\theta\times\frac{\mu^0_{00,00}}{2^\theta}+\sum_{s\neq 1}\sum_y\sum_h\left(\frac{p(\Sigma^s)\frac{q}{\tau^s_{gh}}}{P}\right)^\theta\times\frac{\mu^0_{00,00}}{2^\theta}=L \tag{6}$$

Suppose that the number of unmatched agents weakly increases such that $(\mu^0_{00,00})' \geq \mu^0_{00,00}$. From the previous proof, we know that $p(\Sigma^s)' > p(\Sigma^s)$. Hence it must be that $\left(\frac{p(\Sigma^1)}{\tau^1_{gh}}\right)' < \frac{p(\Sigma^1)}{\tau^1_{gh}}$ $\forall h \neq g$ to satisfy equation (6). However, this is a contradiction. The total value of good 1 and the total value of good s can be expressed as follows:

$$\frac{p(\Sigma^1)^{1-\sigma}}{P^{1-\sigma}}E = \underbrace{\sum_x\sum_g\sum_y\sum_h p(\Sigma^1)\mu^1_{xg,yh}(\mu^0_{xg,00},\mu^0_{00,yh})q(I_{xg},J_{yh},\Sigma^1,\tau^1_{gh})}_{\text{World total value in sector 1}}$$
$$=\sum_x\sum_g\sum_y\sum_h\left(\frac{p(\Sigma^1)\frac{q}{\tau^1_{gh}}}{P}\right)^\theta\times\frac{\mu^0_{00,00}}{2^\theta}\times p(\Sigma^1)\frac{q}{\tau^1_{gh}}.$$

$$\frac{p(\Sigma^s)^{1-\sigma}}{P^{1-\sigma}}E = \underbrace{\sum_x\sum_g\sum_y\sum_h p(\Sigma^s)\mu^s_{xg,yh}(\mu^0_{xg,00},\mu^0_{00,yh})q(I_{xg},J_{yh},\Sigma^s,\tau^s_{gh})}_{\text{World total value in sector 1}}$$
$$=\sum_x\sum_g\sum_y\sum_h\left(\frac{p(\Sigma^s)\frac{q}{\tau^s_{gh}}}{P}\right)^\theta\times\frac{\mu^0_{00,00}}{2^\theta}\times p(\Sigma^s)\frac{q}{\tau^s_{gh}}.$$

Since the relative price of good 1 decreases, the relative total value of goods in sector 1 should increase. This implies that $\left(\frac{p(\Sigma^1)}{\tau^1_{gh}}\right)' > p(\Sigma^s)' > p(\Sigma^s)\ \forall h \neq g$. Hence, the number of unmatched agents, $\mu^0_{00,00}$, should decrease.

(iii) In sector 1, the number of within-country matches and income can be represented as follows:

$$\sum_{y}\sum_{h=g}\left(\frac{p(\Sigma^1)\dfrac{q}{\tau^1_{gh}}}{P}\right)^{\theta}\times\frac{\mu^0_{00,00}}{2^\theta},$$

$$w^1_{xg,yg}=\pi^1_{xg,yg}=\frac{1}{2}p(\Sigma^1)\frac{q}{\tau^1_{gg}}.$$

Because $(\mu^0_{00,00})' < \mu^0_{00,00}$ and $p(\Sigma^1)' < p(\Sigma^1)$, the number of matches and income should decrease. In sector 1, the number of within-country matches and income can be represented as follows:

$$\sum_{y}\sum_{h\neq g}\left(\frac{p(\Sigma^1)\dfrac{q}{\tau^1_{gh}}}{P}\right)^{\theta}\times\frac{\mu^0_{00,00}}{2^\theta},$$

$$w^1_{xg,yg}=\pi^1_{xg,yg}=\frac{1}{2}p(\Sigma^1)\frac{q}{\tau^1_{gh}}.$$

Since $\left(\dfrac{p(\Sigma^1)}{\tau^1_{gh}}\right)' > \left(\dfrac{p(\Sigma^1)}{\tau^1_{gh}}\right)$ $\forall h \neq g$, wages and profits should increase. From the labor market clear condition in equation (6), it is easy to derive that the cross-country matches should increase.

(iv) In other sectors $s \neq 1$, the number of matches and income can be represented as follows:

$$\sum_{s\neq 1}\sum_{y}\sum_{h}\left(\frac{p(\Sigma^s)\dfrac{q}{\tau^s_{gh}}}{P}\right)^{\theta}\times\frac{\mu^0_{00,00}}{2^\theta},$$

$$w^s_{xg,yh} = \pi^s_{xg,yh} = \frac{1}{2}p(\Sigma^s)\frac{q}{\tau^s_{gh}}.$$

Because $p(\Sigma^s)' > p(\Sigma^s)$, incomes should increase. However, since $(\mu^0_{00,00})' < \mu^0_{00,00}$, the number of matches is ambiguous. □

6.3.2 Proof of Proposition 2

Proof. (i) Let $\mu^{0A}_{00,00}$ be the number of unmatched agents in country 1 or 2 and $\mu^{0B}_{00,00}$ be the number of unmatched agents in country 3. The labor market clearing condition can be expressed as follows:

$$\mu^{0A}_{00,00}+\sum_s\sum_{y=1}\sum_h \mu^s_{xg,yh}(\mu^{0A}_{00,00})+\sum_s\sum_{y=2}\sum_h \mu^s_{xg,yh}(\mu^{0A}_{00,00})+\sum_s\sum_{y=3}\sum_h \mu^s_{xg,yh}(\mu^{0A}_{00,00},\mu^{0B}_{00,00}) = L \tag{7}$$

$$\begin{aligned}&\mu^{0B}_{00,00} + \sum_s\sum_{y=1}\sum_h \mu^s_{xg,yh}(\mu^{0A}_{00,00},\mu^{0B}_{00,00}) + \sum_s\sum_{y=2}\sum_h \mu^s_{xg,yh}(\mu^{0A}_{00,00},\mu^{0B}_{00,00}) \\ &+ \sum_s\sum_{y=3}\sum_h \mu^s_{xg,yh}(\mu^{0B}_{00,00}) = L\end{aligned} \tag{8}$$

Since $\tau^s_{12} = \tau^s_{21} = \tau$ to $(\tau^s_{12})' = (\tau^s_{21})' < \tau \ \forall s$, the labor market condition in equation (7) dictates that either $\mu^{0A}_{00,00} < (\mu^{0A}_{00,00})'$ or $\mu^{0B}_{00,00} < (\mu^{0B}_{00,00})'$.

Suppose that $\mu^{0A}_{00,00} < (\mu^{0A}_{00,00})'$. Then, it must be that $\mu^s_{xg,yh}(\mu^{0A}_{00,00},\mu^{0B}_{00,00}) > \left(\mu^s_{xg,yh}(\mu^{0A}_{00,00},\mu^{0B}_{00,00})\right)'$, which implies that $\mu^{0B}_{00,00} > (\mu^{0B}_{00,00})'$. However, this contradicts equation (8). Hence, $\mu^{0A}_{00,00} > (\mu^{0A}_{00,00})'$ and $\mu^{0B}_{00,00} < (\mu^{0B}_{00,00})'$.

(ii) Since $\mu^{0B}_{00,00} < (\mu^{0B}_{00,00})'$, it must be that $\mu^s_{xg,yh}(\mu^{0B}_{00,00}) < \left(\mu^s_{xg,yh}(\mu^{0B}_{00,00})\right)'$. Both inequalities imply that $\mu^s_{xg,yh}(\mu^{0A}_{00,00},\mu^{0B}_{00,00}) > \left(\mu^s_{xg,yh}(\mu^{0A}_{00,00},\mu^{0B}_{00,00})\right)'$. In countries 1 and 2, since $\mu^{0A}_{00,00} > (\mu^{0A}_{00,00})'$ the number of within-country matches should decrease $\mu^s_{xg,yh}(\mu^{0A}_{00,00}) > \left(\mu^s_{xg,yh}(\mu^{0A}_{00,00})\right)'$ while the number of cross-country matches between country 1 and 2 should increase $\mu^s_{xg,yh}(\mu^{0A}_{00,00}) < \left(\mu^s_{xg,yh}(\mu^{0A}_{00,00})\right)'$.

(iii) - (v) Because within-country matching costs do not change and workers and entrepreneurs are symmetric, wages (salaries) do not change in within-country matches:

$$w^s_{xg,yg} = \pi^s_{xg,yg} = \frac{1}{2} p(\Sigma^s) \frac{q}{\tau} \quad \forall s \in \mathcal{S}, x \in \mathcal{X}, y \in \mathcal{Y}, \text{ and } g \in \mathcal{G}.$$

Since $\tau^s_{12} = \tau^s_{21} = \tau$ to $(\tau^s_{12})' = (\tau^s_{21})' < \tau \; \forall s$ and the symmetry of workers and entrepreneurs, wages (salaries) increase in cross-country matching between country 1 and 2:

$$w^s_{x1,y2} = \pi^s_{x1,y2} = \frac{1}{2} p(\Sigma^s) \frac{q}{\tau^s_{12}} \text{ and } w^s_{x2,y1} = \pi^s_{x2,y1} = \frac{1}{2} p(\Sigma^s) \frac{q}{\tau^s_{21}} \quad \forall s \in \mathcal{S}, x \in \mathcal{X}, y \in \mathcal{Y}.$$

Because $\mu^{0A}_{00,00} > (\mu^{0A}_{00,00})'$ and $\mu^{0B}_{00,00} < (\mu^{0B}_{00,00})'$, wages (salaries) increase for agents in country 1 and 2 and wages (salaries) decrease for agents in country 3 in cross-country matching between country 1 (or 2) and 3.

$$w^s_{x1,y3} = w^s_{x2,y3} = \pi^s_{x3,y1} = \pi^s_{x3,y2} = p(\Sigma^s) \frac{q}{\tau} \frac{(\mu^{0A}_{00,00})^{-\frac{1}{\theta}}}{(\mu^{0A}_{00,00})^{-\frac{1}{\theta}} + (\mu^{0B}_{00,00})^{-\frac{1}{\theta}}} \quad \forall s \in \mathcal{S}, x \in \mathcal{X}, y \in \mathcal{Y}.$$

$$w^s_{x3,y1} = w^s_{x3,y2} = \pi^s_{x1,y3} = \pi^s_{x2,y3} = p(\Sigma^s) \frac{q}{\tau} \frac{(\mu^{0B}_{00,00})^{-\frac{1}{\theta}}}{(\mu^{0A}_{00,00})^{-\frac{1}{\theta}} + (\mu^{0B}_{00,00})^{-\frac{1}{\theta}}} \quad \forall s \in \mathcal{S}, x \in \mathcal{X}, y \in \mathcal{Y}.$$

□

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