Offshoring, Matching, and Income Inequality

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Abstract

This paper develops a matching framework of offshoring in which offshoring is defined as a cross-country matching between workers and managers with complementary production technology. We study the distributional effects of offshoring by embedding the matching framework in a two-country, two-task model in which workers and managers possess a continuum of skills. Offshoring alters matching mechanism and influences inequality through differential distributional impacts between-task and within-task in each country. We then introduce an endogenous task choice into the model and show that offshoring changes occupational choice mechanism, which feeds through into distributional consequences. We also extend the model to allow for two industries with endogenous task choice and illustrate that offshoring can happen even when relative output prices are identical between the two countries.

Keywords: Offshoring; Matching; Inequality.
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1 Introduction

With recent reductions in transportation costs and communication costs, production processes have become closely intertwined across countries. In the U.S., the number of foreign manufacturing workers working abroad for U.S. multinationals (using the BEA data) grew from 4.3 million in 2002 to 4.9 million in 2013 (a 13.8% increase) while the total number of U.S. manufacturing workers (using the BLS data) fell from 15.0 to 12.0 million during the same period (a 19.8% decrease). This trend implies that many manufacturing tasks in the U.S. have been sent to developing countries such as Brazil, China, Mexico, and India, which has led to increasing public concern about offshoring U.S. jobs. In particular, much of the debate has been around the extent to which offshoring has led to rising inequality in the U.S. Then, what are the distributional impacts of offshoring?

In this paper, we develop a matching framework of offshoring and derive analytical results regarding the distributional impact of offshoring. Our approach, which is fundamentally derived from a complementary production, extends Grossman, Helpman and Kircher (2017) and Eeckhout and Kircher (2018)'s matching framework to allow for cross-country matching between two countries following Kremer and Maskin (1996, 2006) and Antràs, Garicano and Rossi-Hansberg (2006). The main innovation in this paper is to derive several novel analytical results that shed light on the consequences of offshoring by applying the monotone comparative statics technique as in Costinot and Vogel (2010).

Offshoring is defined as cross-country matching between workers and managers. To analyze the distributional impacts of offshoring, we first study two closed economies with no cross-country matching between workers and managers. Next, we study an integrated world economy in which frictionless cross-country matching is allowed between workers and managers. Then we compare two equilibria with that of an integrated world economy. Specifically, we consider three cases of cross-country differences, each of which provides novel insights into the distributional impact of offshoring. These are: a) *factor endowments*, b) *factor distributions*, and c) *technology levels*.

First, we examine the impact of offshoring under cross-country differences in *factor endowments*, all else equal between two countries. We prove that offshoring strictly increases total production in the world economy; i.e., there are gains from offshoring. On top of that, we demonstrate that total earnings in each country strictly increases from offshoring. This result revives the idea of gains from trade in both countries, as in David Ricardo and Heckscher-Ohlin. However, unlike the representative agent model, our model obtains the result of gains from offshoring in both countries even in the context of our two-sided heterogeneous agent model. At the same time, we show that offshoring generates rising within-country inequality, such that a subset of workers is harmed. In short, abundant factors gain while scarcer factors lose as in Stolper and Samuelson (1941) theorem. We derive this classical result even without assuming international trade, which demonstrates that offshoring mechanism and international trade are different. Furthermore, the workers' share and the managers' share in each country change in response to offshoring, even though we assume a Cobb-Douglas type parameter in production technology.

Second, we investigate the impact of offshoring under cross-country differences in *factor distributions*, all other things being equal between two countries. We conceptualize cross-country differences in *factor distributions* in two perspectives: 1) Centrality - "There are relatively more high-skilled workers/managers in the North than in the South" and 2) Dispersion - "Worker/Manager skill distribution is more diverse in the North than in the South." The underlying mechanism, which pieces together all possible cases, is that cross-country differences in *factor distributions* generate cross-country differences in the matching function (a function that maps from a worker skill to a manager skill) between two countries. Suppose that there are two managers with the same skill level, one in the North and the other in the South, and assume that the Northern manager is paired up with lower-skilled workers than the Southern manager in Autarky. If frictionless cross-country matching is allowed, then the Northern manager will desire to match with Southern workers, which will entail a change in the matching function. Due to the complementary effect, salaries of Northern managers bid up while wages of Northern workers decrease, which generates inequality between managers and workers in the North (between-task inequality). Moreover, more skilled managers benefit more from the re-matching, which generates a skill premium within managers in the North (within-task inequality). In this case, the model explains three salient features of income inequality patterns in the U.S. since the 1960s: the divergence of upper-tail inequality and lower-tail inequality, rising top 1 percent income share, and the growing importance of occupations (see Autor, Katz and Kearney, 2008; Kopczuk, Saez and Song, 2010; Piketty and Saez, 2003; Acemoglu and Autor, 2011).

Third, we study the distributional impact of cross-country differences in *technology levels*, all other things being equal between two countries. We assume that the North is technologically advanced relative to the South with regard to *technology levels*, and we analyze matching patterns and distributional consequences in the South if the South can adopt the advanced Northern technology. One interesting mechanism is that the optimal firm size changes from the technology transfer; in other words, the size of the larger firm increases whereas the size of the smaller firm decreases (the rise of superstar firms), which generates an inequality implication.

Then, we relax the assumption of exogenous supplies of managers and workers and study the impact of offshoring with endogenous task choice under cross-country differences in agent skill distribution. We consider a case in which there are relatively more high-skilled agents in the North than in the South. Offshoring changes the occupational choice mechanism such that some of the most skilled workers become managers and some of the least skilled managers become workers. The matching patterns from offshoring are more complicated than ones with exogenous supplies of managers and workers. We categorize possible results in four cases. Interestingly, in one specific situation, the matching functions in both countries shift upward, implying that all workers match with better managers, and hence it increases within-worker inequality in both countries.

Lastly, we extend the model to allow for two industries with (and without) endogenous task choice. Given the two sectors, each country can gain from international trade if relative output prices are different. This suggests that there would be no international trade if relative output prices are the same between the two countries. We investigate a case in which the relative output prices are identical between the two countries while factor distributions are different across countries. We demonstrate that matching functions in both countries are not the same. Hence, there does not exist international trade between the two countries, while the countries can still gain from offshoring.

2 Related Literature

This paper contributes to the theory of offshoring. Feenstra and Hanson (1996) develop an offshoring model in which a single manufactured good is produced from a continuum of intermediate inputs using skilled workers, unskilled workers, and capital. Grossman and Rossi-Hansberg (2008) propose a task-based offshoring model in which a continuum of tasks is performed by skilled workers and a continuum of tasks is performed by unskilled workers. They explicitly distinguish "goods" and "tasks." Both frameworks analyze the distributional impacts of offshoring when activities/tasks are transferred from North to South. Grossman and Rossi-Hansberg (2012) further explore the task-based offshoring model by extending it to a context of North-North offshoring in which two countries have identical relative factor endowments and technology levels but can differ in size. Yet none of these models allow for the complementary effects between tasks that can arise from the offshoring process (i.e., log-supermodular production technology), and our model explicitly allows for such complementarity. Furthermore, because our model is tractable enough to study both cases, we analyze both North-South offshoring and North-North offshoring.

A few notable exceptions incorporate the complementary effect into the offshoring model. Kremer and Maskin (1996, 2006), who conceptualize globalization as workers from different countries who work together in the same firm, analyze the wage impacts of globalization. Globalization enables high-skilled workers in the developing country to match with more skilled workers in the developed country, while low-skilled workers in the developing country are marginalized because their skill levels are too low to match with workers in the developed country - which generates inequality in the developing country. However, this primary result is derived from a specific case in which the skill level of low-skilled workers in the developing country is too low from the model's strong assumption. Antràs, Garicano and Rossi-Hansberg (2006) propose a knowledge-based hierarchy model to study the effect of cross-country team formation on the structure of wages. Our model is closely related to this study but ours differs in several dimensions. First, in our model the team production function between workers and managers is general while in theirs it is derived from agent's specializations in production and knowledge (Garicano, 2000). Second, we do not impose any particular functional forms for the distributions of skills, which enables us to derive analytical results in more general case. Third, the span-of-control is endogenously determined in our framework.

Concerning the modeling framework, we build upon Grossman, Helpman and Kircher (2017)'s theoretical model that examines the distributional impacts of international trade in a world that has two industries and two factors of production with heterogeneous skill levels. We extend their modeling framework to allow for cross-country matching. In an innovation distinct from Grossman, Helpman and Kircher (2017)'s model we use the concept of likelihood ratio property, as in Milgrom (1981) and Costinot and Vogel (2010), to derive analytical results on the distributional impacts of offshoring. We borrow tools and techniques developed in Costinot and Vogel (2010), whose analysis is restricted to a particular case in which workers only differ in their skills. But going beyond the work of Costinot and Vogel (2010) we apply their techniques to the complementary production function between tasks, which generates different implications for inequality.

Our model also is closely related to the standard two-sided one-to-one matching model originally developed by Becker (1973). More recently, Tervio (2008) has presented a one-to-one matching model between individual managers who have different abilities and firms that have different sizes. However, our paper is distinct from the one-to-one matching model in that we allow for many-to-one matching, and this enables us to speak about the size of the firm that is determined endogenously in the model. In this sense, the model is close to the span-of-control model proposed by Lucas (1978) wherein one manager manages a homogeneous workforce with diminishing returns to scale. Quite recently, Eeckhout and Kircher (2018) have proposed a unifying theoretical model of the many-to-one matching model that studies sorting and firm size simultaneously. Our conceptual framework is most closely related to this structure, and we incorporate the cross-country matching idea into the standard matching theory.

3 The Matching Model of Offshoring

The model builds upon Grossman, Helpman and Kircher (2017) and Eeckhout and Kircher (2018)'s matching framework. We extend their modeling framework to allow for two countries to study the distributional effects of *offshoring*, which we define as a cross-country matching between workers and managers.

3.1 Environment

There are two countries, North (N) and South (S), in the world.[1] In each country, there are $\bar{M}$ units of inelastic "managers" and $\bar{L}$ units of inelastic "workers".[2] Managers are indexed by their skill level $z_M \in \mathcal{M} \subset \mathbb{R}_{++}$, and workers are indexed by their skill level $z_L \in \mathcal{L} \subset \mathbb{R}_{++}$. $\phi_M(z_M)$ is a probability density function over manager skill z_M and $\phi_L(z_L)$ is a probability density function over worker skill z_L. The probability density functions, $\phi_M(z_M)$ and $\phi_L(z_L)$, are both continuous and strictly positive over their bounded supports, $\mathcal{M} = [z_{M,min}, z_{M,max}]$ and $\mathcal{L} = [z_{L,min}, z_{L,max}]$, where $z_{M,min}$, $z_{M,max}$, $z_{L,min}$, and $z_{L,max}$ denote the lowest skill level of managers, the highest skill level of managers, the lowest skill level of workers, and the highest skill level of workers, respectively. Similarly, $\Phi_M(z_M)$ is a cumulative distribution function for manager skill z_M with continuous support $\mathcal{M} = [z_{M,min}, z_{M,max}]$ and $\Phi_L(z_L)$ is a cumulative distribution function for worker skill z_L with continuous support $\mathcal{L} = [z_{L,min}, z_{L,max}]$.

Market structure is perfect competition in which goods are produced by a large number of identical price-taking firms that can freely enter the market. There is only one final good Y in the market whose price P_Y is normalized to one. Each firm hires a "manager" of skill $z_M \in \mathcal{M}$ and some endogenous number of "workers" of skill $z_L \in \mathcal{L}$ to produce final good Y (many-to-one matching). Specifically, we use $m(z_L)$, a matching function, to denote a worker of skill level z_L's counterpart manager skill level. Also, $m^{-1}(z_M)$ is

[1] Section 3 characterizes the closed economy in the North. The South is defined analogously.

[2] More precisely, there are $\bar{M}^N$ units of inelastic "managers" and $\bar{L}^N$ units of inelastic "workers" in the North. In the South, there are $\bar{M}^S$ units of inelastic "managers" and $\bar{L}^S$ units of inelastic "workers." We use superscript N and S to denote countries in an open economy analysis.

an inverse matching function to denote a manager of skill level z_M's counterpart worker skill level.

3.2 Technology

The production function, $F : \mathbb{R}^3_+ \to \mathbb{R}_+$, describes the technology in the economy such that a firm combines manager and workers to produce output Y. A firm hiring one manager of skill level z_M paired up with some endogenous number of workers N of the same skill level z_L can produce a final good Y.[3] The production function in the economy is defined as follows:

$$Y = F(z_M, z_L, N) = \alpha\psi(z_M, z_L)N^\gamma = \alpha e^{z_M^\beta z_L} N^\gamma, \quad \alpha \in \{0, 1\}, \quad \beta > 1, \quad 0 < \gamma < 1$$

where α is the matching technology parameter and N is the number of workers.

Because the cross-country monitoring cost and the coordinating cost are higher than the within-country ones, international matching is less efficient than domestic matching. The technology parameter α captures the international matching friction such that moving from (complete) autarky to (complete) globalization can be modeled as an increase from $\alpha = 0$ to $\alpha = 1$.[4]

$\psi(z_M, z_L)$ denotes the productivity of a production team with one manager of skill z_M and some workers of skill z_L. We specify the following functional form for the productivity of a production team, $e^{z_M^\beta z_L}$, which satisfies following five properties: (i) Managers of different skills are imperfect substitutes for one another, (ii) Workers of different skills are imperfect substitutes for one another, (iii) Different tasks within a firm are complementary, (iv) Different tasks within a firm are differentially sensitive to skill, and (v) The manager's skill and the worker's skill are log supermodular.

Based on the production function in Kremer and Maskin (1996), $z_M^2 z_L$, that captures four main ingredients from (i) to (iv), we add to it log-supermodular property as in (v). A departure from Kremer and Maskin (1996)'s asymmetric and supermodular production function generates different matching patterns. As is well known in the assignment literature, log-supermodularity guarantees a positive assortative matching pattern (Costinot and Vogel, 2010; Sampson, 2014) while the asymmetric and supermodular production can

[3] In Appendix C, we study a case in which a firm can hire some number of managers of the skill level z_M paired up with some number of workers of the skill level z_L (many-to-many matching), and find that the case of the many-to-one matching and the many-to-many matching are identical.

[4] Alternatively, an increase in the technology parameter α can be interpreted as a Hicks-neutral technology change in which the marginal productivity of the worker and the marginal productivity of the manager increase by the same proportion as the technology changes.

generate the "cross-matching" pattern.[5] Lastly, the parameter γ captures the diminishing returns to worker size from the manager's increasing span of control, as in Lucas (1978).

3.3 Profit Maximization

Consider a firm hires a manager of skill z_M. Given the output price $P_Y = 1$ and the wage schedule $w(z_L)$, the firm chooses the skill level of its workers z_L and the number of workers N to maximize profits, including salary payment to the manager:

$$\pi(N, z_L; z_M) = \alpha e^{z_M^\beta z_L} N^\gamma - w(z_L)N \tag{1}$$

where $w(z_L)$ is the wage paid to workers of skill z_L. Differentiating with respect to N yields the conditional worker demand:

$$N(z_L; z_M) = \left[\frac{\gamma \alpha e^{z_M^\beta z_L}}{w(z_L)}\right]^{1/1-\gamma} \tag{2}$$

which represents the optimal number of workers the firm would hire given that the firm hires a manager of skill level z_M, chooses workers of skill level z_L, and faces the wage schedule $w(z_L)$. Then plugging the conditional worker demand $N(z_L; z_M)$ into the profit function in (1) and calculating the first order condition with respect to z_L yields,

$$\frac{z_M^\beta}{\gamma} = \frac{w'(z_L)}{w(z_L)} \tag{3}$$

which shows the firm's optimal choice of worker skill z_L given the manager skill level z_M and the optimal number of workers N. The left-hand side represents the elasticity of productivity with respect to worker skill divided by the parameter γ. The right-hand side denotes the elasticity of wage with respect to worker skill. The first order condition shows the trade-off relationship between productivity and wage: hiring more high-skilled workers bids up productivity, although firms should pay more wages to the more high-skilled workers.[6]

A matching function in this economy is defined as $z_M = m(z_L)$ where $z_M \in \mathcal{M}$ and

[5] Kremer (1993) and Grossman and Maggi (2000) show that the symmetric and supermodular production function exhibits positive assortative matching. Kremer and Maskin (1996)'s production framework is asymmetric and supermodular so that both "cross-matching" and "self-matching" are both allowed in their model.

[6] The same condition can be found in Costinot and Vogel (2010), Sampson (2014), and Grossman, Helpman and Kircher (2017).

$z_L \in \mathcal{L}$. In equilibrium, there exists a unique value z_M that solves (3) for every z_L. Furthermore, the equilibrium exhibits positive assortative matching (PAM) for the economy as a whole.

Proposition 1. (Positive assortative matching) *In equilibrium, the matching function $m(z_L)$ is strictly increasing for all $z_L \in \mathcal{L}$.*

Proof. See Appendix A.1. □

Using the equilibrium matching function $m(z_L)$, the first order condition in (3) can be expressed as:

$$\frac{m(z_L)^\beta}{\gamma} = \frac{w'(z_L)}{w(z_L)}, \quad \text{for all } z_L \in \mathcal{L}. \tag{4}$$

Proposition 2. (Convex log wage schedule) *The log wage schedule $\ln w(z_L)$ is strictly increasing and convex in worker skills.*[7]

Proof. See Appendix A.2. □

Next, let us characterize the salary schedule $r(z_M)$ for managers. If a firm hires a manager of skill z_M and pays him the salary $r(z_M)$, its net profit $\Pi(z_M)$ would be:

$$\Pi(z_M) = \tilde{\pi}(z_M) - r(z_M), \quad \text{for all } z_M \in \mathcal{M} \tag{5}$$

where $\tilde{\pi}(z_M) \equiv \max_{N,z_L} \pi(N, z_L; z_M)$ is the optimal profit including salary payment which is achieved by the choice of the number of workers N and their skill level z_L from (2) and (3). Since the market is perfectly competitive and firms can freely enter the market (free entry condition), all firms in the market earn zero profits $\Pi(z_M) = 0$. Using the zero-profit condition, the following expression can be found:

$$r(z_M) = \gamma^{\gamma/1-\gamma}[1-\gamma]\alpha^{1/1-\gamma}\left[e^{z_M^\beta m^{-1}(z_M)}\right]^{1/1-\gamma} w(m^{-1}(z_M))^{-\gamma/1-\gamma}, \quad \text{for all } z_M \in \mathcal{M}. \tag{6}$$

Differentiating the above expression (6) with respect to z_M yields,

$$\frac{\beta z_M^{\beta-1} m^{-1}(z_M)}{1-\gamma} = \frac{r'(z_M)}{r(z_M)}, \quad \text{for all } z_M \in \mathcal{M} \tag{7}$$

[7] Mincer (1958, 1974) first modeled the log wage schedule as the sum of a linear function of years of education and a quadratic function of years of experience, known as "The Mincer earnings function." Some empirical studies note that log wages are an increasingly convex function of years of education (Lemieux, 2006, 2008). If the skill level is captured well by years of education, the equilibrium convex log wage schedule matches well with the empirical findings. Also note that because the logarithmically convex function implies convex function, the wage schedule $w(z_L)$ also is strictly increasing and convex in worker skills.

where $m^{-1}(z_M)$ is the inverse matching function.[8] Similar to the first order condition in equation (3), the left-hand side of equation (4) represents the elasticity of productivity with respect to manager skill divided by the parameter $1-\gamma$. The right-hand side denotes the elasticity of salary with respect to manager skill.

Proposition 3. (Convex log salary schedule) *The log salary schedule $\ln r(z_M)$ is strictly increasing and convex in manager skills.*[9]

Proof. See Appendix A.3. □

3.4 Factor Market Clearing

Consider any connected set of workers $[z_{La}, z_L]$ and the set of managers $[m(z_{La}), m(z_L)]$ that match with these workers in equilibrium. A manager of skill z_M is matched with $\left[\frac{\gamma\alpha e^{z_M^\beta z_L}}{w(z_L)}\right]^{1/1-\gamma}$ workers of skill z_L. Since the matching function is increasing, factor market clearing condition can be expressed as:

$$\bar{M}\int_{m(z_{La})}^{m(z_L)}\left[\frac{\gamma\alpha e^{z^\beta m^{-1}(z)}}{w(m^{-1}(z))}\right]^{1/1-\gamma}\phi_M(z)dz = \bar{L}\int_{z_{La}}^{z_L}\phi_L(z)dz$$

where the left-hand side is the demand for workers by managers with a skill level between $m(z_{La})$ and $m(z_L)$ and the right-hand side is the supply of workers matched with those managers. After differentiating the factor market clearing condition with respect to z_L, we can derive a differential equation for the matching function as follows:

$$\bar{M}m'(z_L)\left[\frac{\gamma\alpha e^{m(z_L)^\beta z_L}}{w(z_L)}\right]^{1/1-\gamma}\phi_M(m(z_L)) = \bar{L}\phi_L(z_L), \quad \text{for all } z_L \in \mathcal{L}. \tag{8}$$

3.5 Equilibrium

Definition 1. (Competitive equilibrium) *A competitive equilibrium is characterized by a set of functions $m : \mathcal{L} \to \mathcal{M}$, $w : \mathcal{L} \to \mathbb{R}_{++}$, and $r : \mathcal{M} \to \mathbb{R}_{++}$ such that*

[8] Because the equilibrium matching function $m(\cdot)$ shows positive assortative matching, the matching function $m(\cdot)$ is invertible.

[9] Like the wage schedule $w(z_L)$, the salary schedule $r(z_M)$ also is strictly increasing and convex in manager skills. Note also that convexities of the log wage function and the log salary function are guaranteed by the assumption of $\beta > 1$. More generally, if the production function is $\alpha e^{z_M^\beta z_L^\delta}N^\gamma$, then the parameter restriction, "$\beta \geq 1$ and $\delta \geq 1$," is sufficient to yield the convex log wage schedule and the convex log salary schedule.

i) Optimality : Firms maximize profits that satisfy equations (4) and (7),

ii) Market Clearing : Factor market clears as in (8).[10]

3.6 Properties of an Equilibrium

3.6.1 Wage Schedule in Equilibrium

In equilibrium, the wage of a worker with skill level z_L is determined by several factors. First, the parameter γ governs the share of total output that goes to workers. Differentiating the profit in equation (1) with respect to N, we can derive the following result:

$$\underbrace{Nw(z_L)}_{\text{Workers' Share}} = \gamma \underbrace{\alpha e^{m(z_L)^\beta z_L} N^\gamma}_{\text{Total Output}}.$$

Next, rearranging the factor market clearing condition in (8), the equilibrium log wage schedule $\ln w(z_L)$ can be expressed as follows:

$$\ln w(z_L) = \ln\gamma + \ln\alpha + \underbrace{m(z_L)^\beta z_L}_{\text{Matching Effect}} + \underbrace{(1-\gamma)\ln\left[\frac{m'(z_L)\bar{M}\phi_M(m(z_L))}{\bar{L}\phi_L(z_L)}\right]}_{\text{Factor Intensity Effect}}. \tag{9}$$

The above log wage equation (9) indicates that there is a one-to-one relationship between the wage $w(z_L)$ and the matching technology parameter α. A one percent increase in matching technology is associated with a one percent increase in wage. An increase in the parameter β is positively associated with the wage level. The increase in the parameter β is equivalent to manager skill upgrading, and this will, in turn, increase the productivity of the production team. The increased productivity then feeds through the wage level positively. The higher skill level of a matching counterpart, $m(z_L)$, yields a higher wage level. As the productivity of a team $\psi(z_M, z_L)$ is complementary between manager skill and worker skill, a higher level of manager skill will increase the wage of a worker with skill level z_L. Hereafter we denote this effect as a "matching effect." Note that the term in brackets represents the measure of managers to the measure of workers given one unit of skill level z_L. $\frac{d\Phi_M(m(z_L))}{d\Phi_L(z_L)} := \frac{m'(z_L)\phi_M(m(z_L))}{\phi_L(z_L)}$ is called the Radon-Nikodym derivative and it measures the rate of the change of density of the measure of

[10] Note that either the wage schedule $w(z_L)$ or the salary schedule $r(z_M)$ can be recovered from each other due to the zero-profit condition in equation (6). This implies that there are two non-linear ordinary differential equations, (4) and (8). In addition to two equations, we have two boundary conditions from the positive assortive matching property: $z_{M,min} = m(z_{L,min})$ and $z_{M,max} = m(z_{L,max})$. If $\phi_M(z_M)$ and $\phi_L(z_L)$ are continuously differentiable, then the set of functions $m(\cdot)$, $w(\cdot)$, and $r(\cdot)$ are uniquely determined. Furthermore, as the market is complete and competitive, the equilibrium allocation is Pareto optimal.

managers with respect to the measure of workers. Thus, we can interpret the term in the bracket as relative factor intensity at worker skill level z_L. The relative factor intensity is positively related to wage level, which reflects the fact that more workers per production team will reduce the wage level from the competition effect. We will call it the "factor intensity effect" hereafter.

Next, let us examine the within-worker wage inequality in equilibrium. To this end, consider a connected set of workers $[z_{La}, z_{Lb}]$ and a set of managers $[m(z_{La}), m(z_{Lb})]$ that match with these workers. Then, the equilibrium condition can be expressed as follows:

$$\ln w(z_{Lb'}) - \ln w(z_{La'}) = \int_{z_{La'}}^{z_{Lb'}} \frac{m(z)^{\beta}}{\gamma} dz, \quad \text{for all } z_{Lb'} > z_{La'} \text{ and } z_{La'}, z_{Lb'} \in [z_{La}, z_{Lb}] \quad (10)$$

where the left-hand side expression represents the measure of the wage inequality, which is the log difference between the wage of a high-skilled worker and the wage of a low skilled worker. First, the wage inequality is positively associated with the matching function $m(\cdot)$. If the matching function shifts upward for all workers with skill level $z_L \in [z_{La}, z_{Lb}]$, then wage inequality widens. This reflects the fact that the upgrading of the managers skill is beneficial to both low-skilled and high-skilled workers, but the high-skilled workers benefit more because of the complementary effect between manager skill and worker skill. Second, the parameter β also is positively associated with the wage inequality. Since the increase in the parameter β is equivalent to the managers' skill upgrading, the effect is the same as an upward shift of the matching function. Lastly, the parameter γ is negatively associated with the wage inequality. The increase in the parameter γ induces managers to control more workers. This leads workers to match with more able managers. Thus, high-skilled workers are negatively affected by the factor intensity effect while low-skilled workers are made better off by the factor intensity effect.

3.6.2 Salary Schedule in Equilibrium

From the zero profit condition, a manager's share of total output is as follows:

$$\underbrace{r(z_M)}_{\text{Manager's Share}} = (1-\gamma)\underbrace{\alpha e^{m(z_L)^{\beta} z_L} N^{\gamma}}_{\text{Total Output}}.$$

Plugging the equilibrium wage schedule in (9) into the zero-profit condition in (6), we can derive the equilibrium salary schedule as follows:

$$\ln r(z_M) = \ln(1-\gamma) + \ln\alpha + \underbrace{z_M^{\beta} m^{-1}(z_M)}_{\text{Matching Effect}} \underbrace{- \gamma \ln\left[\frac{m'(m^{-1}(z_M))\bar{M}\phi_M(z_M)}{\bar{L}\phi_L(m^{-1}(z_M))}\right]}_{\text{Factor Intensity Effect}}. \tag{11}$$

The matching technology parameter α and the parameter β have the same effects on salary as they did on wage. Like the equilibrium wage schedule, the salary of a manager with skill level z_M is determined by the following forces: the matching effect and the factor intensity effect. A Higher skill level of a matching counterpart, $m^{-1}(z_M)$, generates a higher salary level. However, unlike the wage schedule, relative factor intensity is negatively associated with salary level.

Next, let us investigate the within-manager salary inequality in equilibrium. Consider a connected set of managers $[z_{Ma}, z_{Mb}]$ and a set of workers $[m^{-1}(z_{Ma}), m^{-1}(z_{Mb})]$ that match with these managers. The equilibrium condition can be expressed as:

$$\ln r(z_{Mb'}) - \ln r(z_{Ma'}) = \int_{z_{Ma'}}^{z_{Mb'}} \frac{\beta z^{\beta-1} m^{-1}(z)}{1-\gamma} dz, \quad \text{for all } z_{Mb'} > z_{Ma'} \text{ and } z_{Ma'}, z_{Mb'} \in [z_{Ma}, z_{Mb}] \tag{12}$$

where the left-hand side expression represents the measure of the salary inequality: the log difference between the salary of a high-skilled manager and the salary of a low-skilled manager. Salary inequality is positively associated with the inverse matching function $m^{-1}(\cdot)$. Like the case of the worker's wage, skill upgrading of workers for all managers with skill level $z_M \in [z_{Ma}, z_{Mb}]$ has disproportionate effects on the managers' salary. Similar to the wage inequality, the parameter β is positively associated with the salary inequality. However, unlike the case of the wage inequality, the parameter γ is positively associated with the salary inequality.

3.6.3 Matching Function in Equilibrium

By differentiating the equation (9) with respect to z_L and substituting (4) into the result, we obtain the following second-order differential equation for the matching function:

$$\frac{m''(z_L)}{m'(z_L)} = \frac{m(z_L)^{\beta}}{\gamma} - \frac{\beta m(z_L)^{\beta-1} m'(z_L) z_L}{1-\gamma} + \frac{\phi_L'(z_L)}{\phi_L(z_L)} - \frac{\phi_M'(m(z_L))m'(z_L)}{\phi_M(m(z_L))}. \tag{13}$$

The solution to the above second-order differential equation does not depend on the parameter α and factor endowments $\bar{M}$ and $\bar{L}$. This implies that the matching function $m(z_L)$ does not depend on the parameter α and factor endowments $\bar{M}$ and $\bar{L}$.

4 The Distributional Effects of Offshoring

Let us analyze how offshoring, a cross-country matching between a manager and workers, changes the matching mechanism and thereby affects the distribution of earnings (salary and wage) within and between groups (managers and workers) in the world economy composed of two countries, North and South. In the world economy, managers can match with workers in their own country or with workers in the other country. In within-country matching, we normalize $\alpha = 1$. In this section, we focus on the case of offshoring in which the cross-country matching technology parameter α changes from 0 to 1, or a move from (complete) autarky to (complete) globalization.

The equilibrium in the world economy is analogous to the equilibrium in the closed economy. The difference between the two equilibria lies in the supplies of the managers and workers. The supply of the heterogeneous managers and the supply of heterogeneous workers in the world economy, respectively, are defined by:

$$\bar{M}^W \phi_M^W(z_M) \equiv \bar{M}^N \phi_M^N(z_M) + \bar{M}^S \phi_M^S(z_M), \quad \text{for all } z_M \in \mathcal{M}^W$$

$$\bar{L}^W \phi_L^W(z_L) \equiv \bar{L}^N \phi_L^N(z_L) + \bar{L}^S \phi_L^S(z_L), \quad \text{for all } z_L \in \mathcal{L}^W$$

where $\bar{M}^W \equiv \bar{M}^N + \bar{M}^S$, $\phi_M^W(z_M) \equiv \dfrac{\bar{M}^N}{\bar{M}^N + \bar{M}^S}\phi_M^N(z_M) + \dfrac{\bar{M}^S}{\bar{M}^N + \bar{M}^S}\phi_M^S(z_M)$, $\mathcal{M}^W \equiv \mathcal{M}^N \cup \mathcal{M}^S$, $\bar{L}^W \equiv \bar{L}^N + \bar{L}^S$, $\phi_L^W(z_L) \equiv \dfrac{\bar{L}^N}{\bar{L}^N + \bar{L}^S}\phi_L^N(z_L) + \dfrac{\bar{L}^S}{\bar{L}^N + \bar{L}^S}\phi_L^S(z_L)$, and $\mathcal{L}^W \equiv \mathcal{L}^N \cup \mathcal{L}^S$.[11]

It is instructive to investigate the effects of offshoring under cross-country differences in factor endowments and factor distributions in isolation. In the following subsections, we begin by examining the impact of offshoring under cross-country differences in *factor endowments* given same factor distributions across countries. Next, we consider the effect of offshoring under cross-country differences in *factor distributions* while holding identical factor endowments across countries. Factor distributions differ across countries in terms of centrality and dispersion. For centrality, we use a concept of monotone likelihood ratio property, as in Milgrom (1981) and Costinot and Vogel (2010). For dispersion, we use a concept of diversity of skill, as in Grossman and Maggi (2000) and Costinot and Vogel (2010). In each case, we investigate how the falling costs of offshoring affect the matching function $m(\cdot)$ in each country and conclude its implications for the wage function $w(\cdot)$ and the salary function $r(\cdot)$. Along with the analytical solutions, we provide numerical exercises in each case.

[11]The superscripts W, N, and S denote World, North and South, respectively. The world distribution is a *mixture distribution* of the distribution in the North and the distribution in the South.

4.1 Cross-Country Differences in Factor Endowments

Proposition 4. (Gains from offshoring, global inequality, and within-country inequality) *Suppose that the North and the South are identical, except that there are relatively more managers in the North than in the South,* $\frac{\bar{M}^N}{\bar{L}^N} > \frac{\bar{M}^S}{\bar{L}^S}$. *If the economy moves from (complete) autarky to (complete) globalization, then*

(i) Total production in the world strictly increases;

(ii) Total earnings in each country strictly increases;

(iii) The prices of identical factors of production are equalized;

(iv) In the North, the wage schedule shifts downward while the salary schedule shifts upward. In the South, the wage schedule shifts upward while the salary schedule shifts downward.[12]

Proof. See Appendix A.4. □

The first result illustrates efficiency gains from offshoring caused by the change in matching mechanism between workers and managers that leads to the expansion of total production. Because the within-country matching is always possible in the integrated world economy, we can always replicate the closed economies of North and South in globalization, and competitive market forces re-organize production units efficiently. A social planner who wishes to maximize total production in the world would prefer the equilibrium in globalization to those in the closed economy. The second result shows that both countries are better off from offshoring given that a social planner in each country wants to maximize total earnings in each country. There exists a pattern of lump sum transfers within each country such that all agents can gain from offshoring, even without transfers between countries. The third result shows that after globalization, workers of the same skill level receive the same wages and managers of the same skill level earn the same salaries. This result is consistent with Samuelson (1948)'s "Factor price equalization."

The fourth result reveals that the abundant factor gains while the scarcer factor loses from offshoring, which is similar to the Stolper and Samuelson (1941) theorem (the homogenous workers and homogeneous managers case). Although the results echo earlier

[12] To derive numerical exercise results, we use a bounded Pareto distribution with shape parameter $k_M > 0$ and location parameters $z_{M,min} > 0$ and $z_{M,max} > 0$ for manager skill distribution $\phi_M(z_M)$. Likewise, worker skill distribution $\phi_L(z_L)$ follows a bounded Pareto distribution with shape parameter $k_L > 0$ and location parameters $z_{L,min} > 0$ and $z_{L,max} > 0$. Next, the sensitivity to manager skill parameter β is based on Kremer and Maskin (1996, 2006)'s production function $\psi(z_M, z_L) = z_M^2 z_L$. In our specification, team productivity is $\psi(z_M, z_L) = e^{z_M^\beta z_L}$ and we set $\beta = 1.2$, implying that team productivity ranges from 2.7 (the lowest) to 99.0 (the highest). The span of control parameter γ is taken directly from the model of Atkeson and Kehoe (2005), in which the share of total output paid to the worker is 65.1 percent. In Appendix B - Table 5, we present sets of parameter values that characterize the move from autarky to globalization in Proposition 4. In Appendix B - Figure 9 shows the autarky and open economy matching functions, log wage functions, and log salary functions given in Proposition 4.

work that examined the distributional impacts of international trade, our results achieve Samuelson (1948)'s "Factor price equalization" and Stolper and Samuelson (1941) theorem from different mechanism (i.e., offshoring) because there is no international trade in our model. Quantitatively, a one percent increase in factor endowment $\frac{\bar{M}}{\bar{L}}$ raises the wage schedule $w(z_L)$ by $1-\gamma$ percent for all $z_L \in \mathcal{L}^W$ and reduces the salary schedule $r(z_M)$ by γ percent for all $z_M \in \mathcal{M}^W$.[13] What are the within-country inequality implications from offshoring in this case? If the average salary of managers is higher than the average wage of workers, between-group inequality widens in the North while it narrows in the South. Note, however, that differences in factor endowments do not generate within-group inequality because the matching function does not change. This result also demonstrates that even though the worker share of total output and the manager share of total output are pinned down by the Cobb-Douglas type parameter γ in a closed economy, offshoring can alter the workers' share and the managers' share. Because Northern managers can supervise more workers, including Southern workers, from offshoring, the managers' share rises and the workers' share shrinks in the North.

4.2 Cross-Country Differences in Factor Distributions (Centrality)

Definition 2. (Monotone likelihood ratio property - centrality)

(i) There are relatively more high-skilled managers in the North than in the South if $\frac{\phi_M^N(z'_M)}{\phi_M^N(z_M)} \geq \frac{\phi_M^S(z'_M)}{\phi_M^S(z_M)} \quad \forall z'_M \geq z_M$ *holds.*

(ii) There are relatively more high-skilled workers in the North than in the South if $\frac{\phi_L^N(z'_L)}{\phi_L^N(z_L)} \geq \frac{\phi_L^S(z'_L)}{\phi_L^S(z_L)} \quad \forall z'_L \geq z_L$ *holds.*[14]

[13] When the span of control parameter γ is low, the marginal return of team production from increasing more units of workers diminishes at a higher rate. Thus, workers are more sensitive to the change in relative factor endowment $\frac{\bar{M}}{\bar{L}}$ and this feeds through into the higher response of the wage schedule than that of the salary schedule.

[14] Examples of families of distributions with this property are the Normal (with mean θ), the Exponential (with mean θ), the Poisson (with mean θ), the Uniform (on $[0, \theta]$), and many others. The monotone likelihood ratio property extends the idea of skill abundance in a two-factor model into a continuum of skill framework. If $z_L, z'_L \in \mathcal{L}^N \cap \mathcal{L}^S$, then the monotone likelihood ratio property implies that $\frac{\phi_L^N(z'_L)}{\phi_L^N(z_L)} \geq \frac{\phi_L^S(z'_L)}{\phi_L^S(z_L)}$. The property also encompasses a case where different sets of skills are available under $\mathcal{L}^N$ and $\mathcal{L}^S$. If $z_L, z'_L \notin \mathcal{L}^N \cap \mathcal{L}^S$, then $z_L \in \mathcal{L}^S$ and $z'_L \in \mathcal{L}^N$. In other words, $\mathcal{L}^N$ is greater than $\mathcal{L}^S$ in the stronger set order: $z^N_{L,min} \geq z^S_{L,min}$ and $z^N_{L,max} \geq z^S_{L,max}$. The highest skilled worker is in $\mathcal{L}^N$ and the lowest skilled worker is in $\mathcal{L}^S$.

4.2.1 Cross-Country Differences in Worker Distributions (Centrality)

Lemma 1. *Suppose* $\frac{\phi_L^N(z_L')}{\phi_L^N(z_L)} \geq \frac{\phi_L^S(z_L')}{\phi_L^S(z_L)} \quad \forall z_L' \geq z_L$. *Then,*

(i) $m^N(z_L) \leq m^S(z_L)$ *for all* $\mathcal{L}^N \cap \mathcal{L}^S$*;*

(ii) $\phi_L^W(z_L)$ *satisfies* $\frac{\phi_L^N(z_L')}{\phi_L^N(z_L)} \geq \frac{\phi_L^W(z_L')}{\phi_L^W(z_L)} \geq \frac{\phi_L^S(z_L')}{\phi_L^S(z_L)}$.

Proof. See Appendix A.5. □

Because the relative supply of high skilled workers is more abundant in the North than in the South, Northern workers match with the less skilled manager than Southern workers although the two workers have the same skill level.[15] The second result shows that if the North has more high skilled workers relative to the South, then the North has more high skilled workers relative to the World and the World has more high skilled workers relative to the South.

Proposition 5. *Suppose that the North and the South are identical, except that there are relatively more high skilled workers in the North than in the South,* $\frac{\phi_L^N(z_L')}{\phi_L^N(z_L)} \geq \frac{\phi_L^S(z_L')}{\phi_L^S(z_L)} \quad \forall z_L' \geq z_L$. *If the economy moves from (complete) autarky to (complete) globalization, then*

(i) $m^W(z_L) \geq m^N(z_L) \quad \forall z_L \in \mathcal{L}^N$*;*

(ii) $m^S(z_L) \geq m^W(z_L) \quad \forall z_L \in \mathcal{L}^S$*;*

(iii) In the North, wage inequality widens and salary inequality narrows;

(iv) In the South, wage inequality narrows and salary inequality widens.[16]

Proof. See Appendix A.6. □

From a Northern worker's standpoint, offshoring implies the upgrading of the matching partner's skill. From a Northern manager's perspective, offshoring means the downgrading of matching partner's skill. Intuitively, as the relative supply of low-skilled workers increases in the North, the positive assortative matching (PAM) condition requires that low-skilled managers match with more low-skilled workers in the North.[17]

[15] From a manager's standpoint, the Northern manager can match with higher skilled workers than the Southern manager although the two managers have the same skill level.

[16] In order to model monotone likelihood ratio property in the numerical exercise, $\frac{\phi_L^N(z_L')}{\phi_L^N(z_L)} \geq \frac{\phi_L^S(z_L')}{\phi_L^S(z_L)} \quad \forall z_L' \geq z_L$, we set the Pareto shape parameter values as follows: $k_W^N = 2$ and $k_W^S = 10$. In Appendix B - Table 6, we present sets of parameter values that characterize the move from autarky to globalization in Propostion 5. Appendix B - Figure 10 shows the autarky and open economy matching functions, log wage functions, and log salary functions given in Proposition 5.

[17] In the South, the results are the opposite.

In the North, an increase in the relative supply of the low-skilled workers triggers a matching of all workers toward high-skilled managers. Given that the production function is log-supermodular between the manager's skill and the worker's skill, high-skilled workers in the North benefit more from this re-matching process than low-skilled workers in the North, and this leads to wage inequality among Northern workers.[18]

Equations (10) and (12) indicate that the parameter γ is associated with the sensitivity of wage inequality and salary inequality. When the parameter γ is low, wage inequality reacts sharply while salary inequality responds slowly in response to a change in the matching function. The logic is such that the marginal return of team production from increasing more units of workers diminishes at a higher rate. Unlike the parameter γ, the sensitivity of the manager's skill β is positively associated with both wage inequality and salary inequality.

4.2.2 Cross-Country Differences in Manager Distributions (Centrality)

Lemma 2. *Suppose* $\frac{\phi_M^N(z'_M)}{\phi_M^N(z_M)} \geq \frac{\phi_M^S(z'_M)}{\phi_M^S(z_M)} \quad \forall z'_M \geq z_M$. *Then,*

(i) $m^{N^{-1}}(z_M) \leq m^{S^{-1}}(z_M)$ *for all* $\mathcal{M}^N \cap \mathcal{M}^S$*;*

(ii) $\phi_M^W(z_M)$ *satisfies* $\frac{\phi_M^N(z'_M)}{\phi_M^N(z_M)} \geq \frac{\phi_M^W(z'_M)}{\phi_M^W(z_M)} \geq \frac{\phi_M^S(z'_M)}{\phi_M^S(z_M)}$.

Proof. See Proof of Lemma 1. □

Proposition 6. *Suppose that the North and the South are identical, except that there are relatively more high skilled managers in the North than in the South,* $\frac{\phi_M^N(z'_M)}{\phi_M^N(z_M)} \geq \frac{\phi_M^S(z'_M)}{\phi_M^S(z_M)} \quad \forall z'_M \geq z_M$. *If the economy moves from (complete) autarky to (complete) globalization. Then,*

(i) $m^{W^{-1}}(z_M) \geq m^{N^{-1}}(z_M) \quad \forall z_M \in \mathcal{M}^N$*;*

(ii) $m^{S^{-1}}(z_M) \geq m^{W^{-1}}(z_M) \quad \forall z_M \in \mathcal{M}^S$*;*

(iii) In the North, wage inequality narrows and salary inequality widens;

(iv) In the South, wage inequality widens and salary inequality narrows.

Proof. See Proof of Proposition 5. □

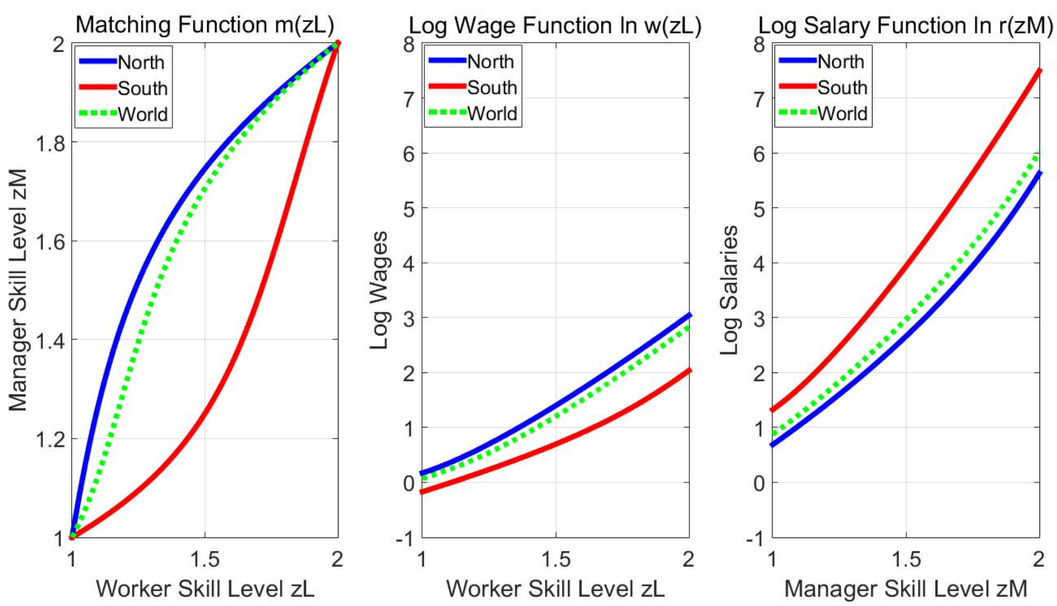
Now, we turn to a case where manager skill distributions differ between the two countries. In Table 1, we present sets of parameter values that characterize the move from autarky to globalization in Propostion 6. Figure 1 shows the autarky and open economy matching functions, log wage functions, and log salary functions given in Proposition 6.

[18] Analogously, all Northern managers now match with lower-skilled workers. Due to the log-supermodularity, high-skilled managers lose more from this re-matching process than low-skilled managers in the North, and this narrows inequality within managers.

Table 1: Sets of parameter values for Proposition 6

Parameter	Description	North	South	World
$\bar{M}$	Number of managers	100	100	200
$\bar{L}$	Number of workers	1,000	1,000	2,000
$\mathcal{M}$	Set of manager skill levels	[1,2]	[1,2]	[1,2]
$\mathcal{L}$	Set of worker skill levels	[1,2]	[1,2]	[1,2]
k_M	Shape parameter for manager skill distribution	**2**	**10**	$\mathbf{k_M \in (2, 10)}$
k_L	Shape parameter for worker skill distribution	2	2	2
β	Sensitivity to manager skill level	1.2	1.2	1.2
γ	Span of control	0.65	0.65	0.65

Figure 1: The impact of offshoring under cross-country differences in manager distribution



This exercise describes factor distributions between the U.S. and the rest of the world. Using a management field experiment on large Indian textile firms, Bloom, Eifert, Mahajan, McKenzie and Roberts (2013) find that the management practices raised the productivity of firms. This result suggests that large productivity differences across countries have their origins in variations in management practices. Since the U.S. has higher productivity than the rest of the world, we can regard North as the U.S. and South as the rest of the world. The distributional consequences of offshoring, in this case, generates some salient features of income inequality in the U.S. since the 1960s: 1) Earnings polarization, 2) Rising top one percent income share, and 3) Task premium between workers and managers.

First, Autor, Katz and Kearney (2008) observe that according to the U.S. CPS data, a 90/50 (upper-tail) residual wage inequality rose while a 50/10 (lower-tail) residual wage inequality fell during the period 1989 to 2005. Similarly, Kopczuk, Saez and Song (2010) show that a 80/50 (upper-tail) earnings ratio among men rose while a 50/20 (lower-tail) earnings ratio among men fell during the 1990s (data from the U.S. Social Security Administration). As managers, on average, are better paid than workers, the upper-tail inequality can be considered as within-manager inequality and the lower-tail inequality can be regarded as within-worker inequality. In the North, within-manager inequality widens and within-worker inequality narrows, which is consistent with the observations of Autor, Katz and Kearney (2008) and Kopczuk, Saez and Song (2010). Second, Piketty and Saez (2003) note that top one percent income shares have humongously risen since the 1970s using the U.S. individual tax returns data. Since top income earners are managers and within-manager inequality widens in the North, the top one percent income rises from globalization. Lastly, Acemoglu and Autor (2011) point out that individuals' tasks have become an important determinant of earnings. The earnings gap between managerial task and subordinate task widened during the period 1973 to 2009 using Census and American Community Survey data. As the matching function shifts downward in the North, Northern managers are matched with higher skilled workers while Northern workers are paired up with lesser skilled managers from globalization. Thus, task premium between managers and workers widens in the North.

4.3 Cross-Country Differences in Factor Distributions (Dispersion)

Definition 3. (Monotone likelihood ratio property - dispersion)

(i) Manager skill distribution is more diverse in the North than in the South if $\frac{\phi_M^N(z'_M)}{\phi_M^N(z_M)} \geq \frac{\phi_M^S(z'_M)}{\phi_M^S(z_M)}$ for all $z'_M \geq z_M \geq \hat{z}_M$, *and* $\frac{\phi_M^N(z'_M)}{\phi_M^N(z_M)} \leq \frac{\phi_M^S(z'_M)}{\phi_M^S(z_M)}$ for all $z_M \leq z'_M < \hat{z}_M$.

(ii) Worker skill distribution is more diverse in the North than in the South if $\frac{\phi_L^N(z'_L)}{\phi_L^N(z_L)} \geq \frac{\phi_L^S(z'_L)}{\phi_L^S(z_L)}$ for all $z'_L \geq z_L \geq \hat{z}_L$, *and* $\frac{\phi_L^N(z'_L)}{\phi_L^N(z_L)} \leq \frac{\phi_L^S(z'_L)}{\phi_L^S(z_L)}$ for all $z_L \leq z'_L < \hat{z}_L$.[19]

4.3.1 Cross-Country Differences in Worker Distributions (Dispersion)

Lemma 3. *Suppose* $\frac{\phi_L^N(z'_L)}{\phi_L^N(z_L)} \geq \frac{\phi_L^S(z'_L)}{\phi_L^S(z_L)}$ for all $z'_L \geq z_L \geq \hat{z}_L$, *and* $\frac{\phi_L^N(z'_L)}{\phi_L^N(z_L)} \leq \frac{\phi_L^S(z'_L)}{\phi_L^S(z_L)}$ for all $z_L \leq z'_L < \hat{z}_L$. *Then,*

[19]These properties capture the idea that there are relatively more managers with extreme skill levels in the North than in the South. The examples of distribution with this property are the Normal (with same mean θ and different variance σ), the Uniform (on $[\theta_1, \theta_2]$ vs. $[\theta_3, \theta_4]$ with $\theta_1 > \theta_3$ and $\theta_2 < \theta_4$), and many others.

(i) There exists $z_L^* \in \mathcal{L}^W$ such that $m^N(z_L) \geq m^S(z_L)$ for all $z_L \in [z^S_{L,min}, z_L^*]$ and $m^N(z_L) \leq m^S(z_L)$ for all $z_L \in [z_L^*, z^S_{L,max}]$;

(ii) $\phi_L^W(z_L)$ satisfies $\frac{\phi_L^N(z_L')}{\phi_L^N(z_L)} \geq \frac{\phi_L^W(z_L')}{\phi_L^W(z_L)}$ for all $z_L' \geq z_L \geq \hat{z}_L$, and $\frac{\phi_L^N(z_L')}{\phi_L^N(z_L)} \leq \frac{\phi_L^W(z_L')}{\phi_L^W(z_L)}$ for all $z_L \leq z_L' < \hat{z}_L$. Also, $\phi_L^W(z_L)$ satisfies $\frac{\phi_L^W(z_L')}{\phi_L^W(z_L)} \geq \frac{\phi_L^S(z_L')}{\phi_L^S(z_L)}$ for all $z_L' \geq z_L \geq \hat{z}_L$, and $\frac{\phi_L^W(z_L')}{\phi_L^W(z_L)} \leq \frac{\phi_L^S(z_L')}{\phi_L^S(z_L)}$ for all $z_L \leq z_L' < \hat{z}_L$.

Proof. See Appendix A.7. □

Because the relative supply of higher skilled workers is larger among the high-skill worker group in the North than in the South, Northern workers in the high-skill worker group match with the less skilled manager than Southern workers in the high-skill worker group. A Northern manager in the high-skill manager group can better match with higher skilled workers than a Southern manager in the high-skill manager group.[20]

Definition 4. (Convergence and polarization of the earnings schedule)

(i) The salary schedule, $r(z_M)$, converges if there is an increase in inequality among low-skilled managers, $z_M < z_M^*$, and a decrease in inequality among high-skilled managers, $z_M > z_M^*$. The salary schedule, $r(z_M)$, polarizes if there is a decrease in inequality among low-skilled managers, $z_M < z_M^*$, and an increase in inequality among high-skilled managers, $z_M > z_M^*$.

(ii) The wage schedule, $w(z_L)$, converges if there is an increase in inequality among low-skilled workers, $z_L < z_L^*$, and a decrease in inequality among high-skilled workers, $z_L > z_L^*$. The wage schedule, $w(z_L)$, polarizes if there is a decrease in inequality among low-skilled workers, $z_L < z_L^*$, and an increase in inequality among high-skilled workers, $z_L > z_L^*$.

Proposition 7. *Suppose that the North and the South are identical, except that worker skill distribution is more diverse in the North than in the South, $\frac{\phi_L^N(z_L')}{\phi_L^N(z_L)} \geq \frac{\phi_L^S(z_L')}{\phi_L^S(z_L)}$ for all $z_L' \geq z_L \geq \hat{z}_L$, and $\frac{\phi_L^N(z_L')}{\phi_L^N(z_L)} \leq \frac{\phi_L^S(z_L')}{\phi_L^S(z_L)}$ for all $z_L \leq z_L' < \hat{z}_L$. If the economy moves from (complete) autarky to (complete) globalization, then*

(i) $m^N(z_L) \geq m^W(z_L) \quad \forall z_L \in [z^N_{L,min}, z_L^{N*}]$ and $m^N(z_L) \leq m^W(z_L) \quad \forall z_L \in [z_L^{N*}, z^N_{L,max}]$;

(ii) $m^W(z_L) \geq m^S(z_L) \quad \forall z_L \in [z^S_{L,min}, z_L^{S*}]$ and $m^W(z_L) \leq m^S(z_L) \quad \forall z_L \in [z_L^{S*}, z^S_{L,max}]$;

(iii) In the North, the wage schedule polarizes and the salary schedule converges;

(iv) In the South, the wage schedule converges and the salary schedule polarizes.[21]

[20] For the low-skill worker group and the low-skill manager group, the result is precisely the opposite.

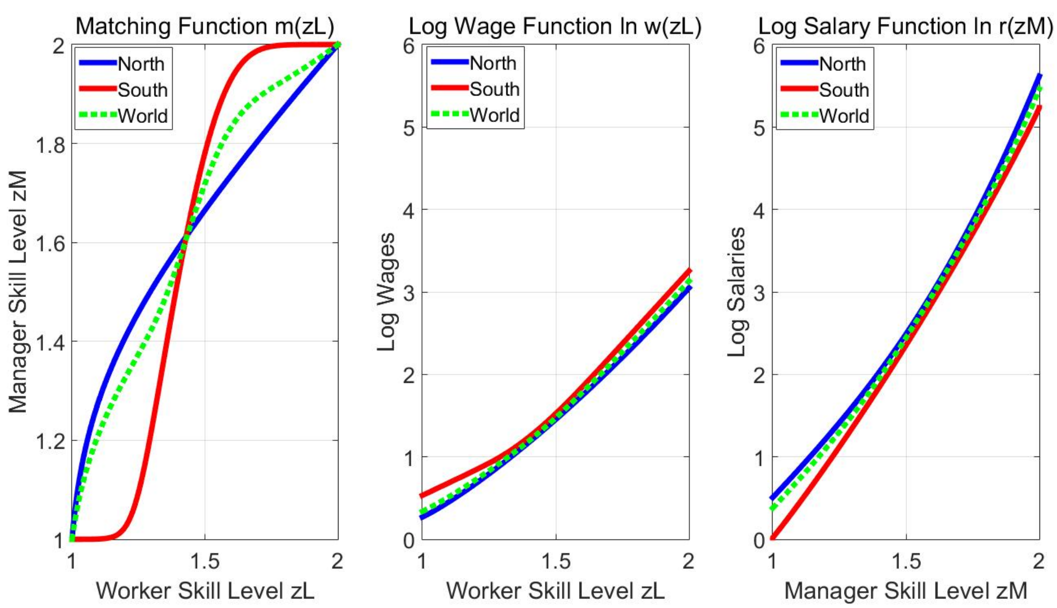
[21] To capture skill dispersion, we use a bounded Normal distribution with mean $\mu_M > 0$, variance $\sigma_M > 0$, and location parameters $z_{M,min} > 0$ and $z_{M,max} > 0$ for manager skill distribution $\phi_M(z_M)$. Likewise, worker skill distribution $\phi_L(z_L)$ follows a bounded Normal distribution with mean $\mu_L > 0$, variance $\sigma_L > 0$, and location parameters $z_{L,min} > 0$ and $z_{L,max} > 0$.

Proof. See Proof of Proposition 5. □

Table 2: Sets of parameter values for Proposition 7

Parameter	Description	North	South	World
$\bar{M}$	Number of managers	100	100	200
$\bar{L}$	Number of workers	1,000	1,000	2,000
$\mathcal{M}$	Set of manager skill levels	[1,2]	[1,2]	[1,2]
$\mathcal{L}$	Set of worker skill levels	[1,2]	[1,2]	[1,2]
μ_M	Mean parameter for manager skill distribution	1.5	1.5	1.5
μ_L	Mean parameter for worker skill distribution	1.5	1.5	1.5
σ_M	Variance parameter for manager skill distribution	0.3	0.3	0.3
σ_L	Variance parameter for worker skill distribution	**0.6**	**0.1**	$\sigma_\mathbf{L} \in (\mathbf{0.1}, \mathbf{0.6})$
β	Sensitivity to manager skill level	1.2	1.2	1.2
γ	Span of control	0.65	0.65	0.65

Figure 2: The impact of offshoring under cross-country differences in worker distribution



In Table 2, we present sets of parameter values that characterize the move from autarky to globalization that Proposition 7 describes. Figure 2 shows the autarky and open economy matching functions, log wage functions, and log salary functions given in Proposition 7.

Offshoring implies an upgrading of the matching partner's skill for a high-skilled group of Northern workers while it means a downgrading of the matching partner's skill for a low-skilled group of Northern workers. From a Northern manager's standpoint, offshoring induces the high-skilled manager group to match with lower skill workers while it leads to an upgrading of the matching partner's skill for the low-skilled manager group.[22]

In the North, the least-skilled worker benefits most from the low-skilled worker group, and the most-skilled worker benefits most from the high-skilled worker group. This leads to *polarization* of wage schedule in the North. The mid-skilled manager group, compared to the high-skilled manager group and the low-skilled manager group, benefits relatively more from globalization, which leads to *convergence* of the salary schedule in the North. Kremer and Maskin (1996) argue that a rise in skill dispersion plus an increase in mean skill level raises the wages of the high-skilled but reduces the wages of the low-skilled, thereby increasing inequality. Our result shows that when worker skill distribution is more diverse in the North than in the South wages in the lower and the upper tails of the worker distribution fall in comparison to the median of the worker distribution because there are relatively more workers in the lower and the upper tails of the worker distribution in the North.

4.3.2 Cross-Country Differences in Manager Distributions (Dispersion)

Lemma 4. *Suppose* $\frac{\phi_M^N(z'_M)}{\phi_M^N(z_M)} \geq \frac{\phi_M^S(z'_M)}{\phi_M^S(z_M)}$ for all $z'_M \geq z_M \geq \hat{z}_M$, *and* $\frac{\phi_M^N(z'_M)}{\phi_M^N(z_M)} \leq \frac{\phi_M^S(z'_M)}{\phi_M^S(z_M)}$ for all $z_M \leq z'_M < \hat{z}_M$. *Then,*

(i) There exists $z_M^* \in \mathcal{M}^W$ *such that* $m^{N^{-1}}(z_M) \geq m^{S^{-1}}(z_M)$ *for all* $z_M \in [z_{M,min}^S, z_M^*]$ *and* $m^{N^{-1}}(z_M) \leq m^{S^{-1}}(z_M)$ *for all* $z_M \in [z_M^*, z_{M,max}^S]$;

(ii) $\phi_M^W(z_M)$ *satisfies* $\frac{\phi_M^N(z'_M)}{\phi_M^N(z_M)} \geq \frac{\phi_M^W(z'_M)}{\phi_M^W(z_M)}$ for all $z'_M \geq z_M \geq \hat{z}_M$, *and* $\frac{\phi_M^N(z'_M)}{\phi_M^N(z_M)} \leq \frac{\phi_M^W(z'_M)}{\phi_M^W(z_M)}$ for all $z_M \leq z'_M < \hat{z}_M$. *Also,* $\phi_M^W(z_M)$ *satisfies* $\frac{\phi_M^W(z'_M)}{\phi_M^W(z_M)} \geq \frac{\phi_M^S(z'_M)}{\phi_M^S(z_M)}$ for all $z'_M \geq z_M \geq \hat{z}_M$, *and* $\frac{\phi_M^W(z'_M)}{\phi_M^W(z_M)} \leq \frac{\phi_M^S(z'_M)}{\phi_M^S(z_M)}$ for all $z_M \leq z'_M < \hat{z}_M$.

Proof. See Proof of Lemma 3. □

Proposition 8. *Suppose that the North and the South are identical, except that manager skill distribution is more diverse in the North than in the South,* $\frac{\phi_M^N(z'_M)}{\phi_M^N(z_M)} \geq \frac{\phi_M^S(z'_M)}{\phi_M^S(z_M)}$ for all $z'_M \geq z_M \geq \hat{z}_M$, *and* $\frac{\phi_M^N(z'_M)}{\phi_M^N(z_M)} \leq \frac{\phi_M^S(z'_M)}{\phi_M^S(z_M)}$ for all $z_M \leq z'_M < \hat{z}_M$. *If the economy moves from (complete) autarky to (complete) globalization, then*

(i) $m^{N^{-1}}(z_M) \geq m^{W^{-1}}(z_M) \;\; \forall z_M \in [z_{M,min}^N, z_M^{N*}]$ *and* $m^{N^{-1}}(z_M) \leq m^{W^{-1}}(z_M) \;\; \forall z_M \in [z_M^{N*}, z_{M,max}^N]$;

[22] In the South, results are the opposite.

(ii) $m^{W^{-1}}(z_M) \geq m^{S^{-1}}(z_M)$ $\forall z_M \in [z^S_{M,min}, z^{S*}_M]$ *and* $m^{W^{-1}}(z_M) \leq m^{S^{-1}}(z_M)$ $\forall z_M \in [z^{S*}_M, z^S_{M,max}]$;

(iii) In the North, the wage schedule converges and the salary schedule polarizes;

(iv) In the South, the wage schedule polarizes and the salary schedule converges.[23]

Proof. See Proof of Proposition 5. □

5 The Distributional Effects of Technology Transfer

Let us investigate how changes in technology parameters α, β, and γ affect the matching function $m(\cdot)$, the wage function $w(\cdot)$, and the salary function $r(\cdot)$. Suppose there are two countries in the world and the North is technologically advanced than the South in terms of different parameter values of α, β, and γ. We assume that only within-country matching is allowed and compare two economies that have different parameter values.[24]

5.1 Hicks-Neutral Technology Transfer

Proposition 9. *Suppose that the South can adopt a Northern technology - i.e., from* $\alpha^S \psi(z_M, z_L) N^\gamma$ *to* $\alpha^{S'} \psi(z_M, z_L) N^\gamma$ *where* $\alpha^{S'} = \alpha^N > \alpha^S$. *Then, in the South,*

(i) The matching function does not change;

(ii) The wage schedule shifts upward and the salary schedule shifts upward.[25]

Proof. See Appendix A.8. □

This exercise illustrates that Southern firms can use superior Northern technology through technology transfer. The technology transfer is modeled as a Hicks-neutral technical progress such that the marginal product of the manager and the marginal product of the worker increase in the same proportion. In addition, the technology transfer does not affect the trade-off relation between the productivity and wage/salary in equations (4) and (7). The Hicks-neutral technology progress does not affect the matching function; thus, the ratio of managers and workers stays the same for all production units. Quantitatively, a one percent increase in technology level α raises $w(z_L)$ and $r(z_M)$ by one percent.

[23] In Appendix B - Table 7, we present sets of parameter values that characterize the move from autarky to globalization as described in Proposition 8. Appendix B - Figure 11 shows the autarky and open economy matching functions, log wage functions, and log salary functions given in Proposition 8.

[24] Throughout the analysis, we assume that factor endowments and factor distributions are identical between North and South.

[25] In Appendix B - Table 8, we present sets of parameter values that characterize Propostion 9. Appendix B - Figure 12 shows North and South matching functions, log wage functions, and log salary functions given in Proposition 9.

5.2 Improvement in Management Technology

Proposition 10. *Suppose that the South can adopt a Northern management technology - i.e., from $\alpha^S\psi(z_M, z_L)N^{\gamma^S}$ to $\alpha^S\psi(z_M, z_L)N^{\gamma^{S'}}$ such that $\gamma^{S'} = \gamma^N > \gamma^S$. Then, in the South,*

(i) The matching function shifts upward;

(ii) The workers' share increases and the managers' share reduces;

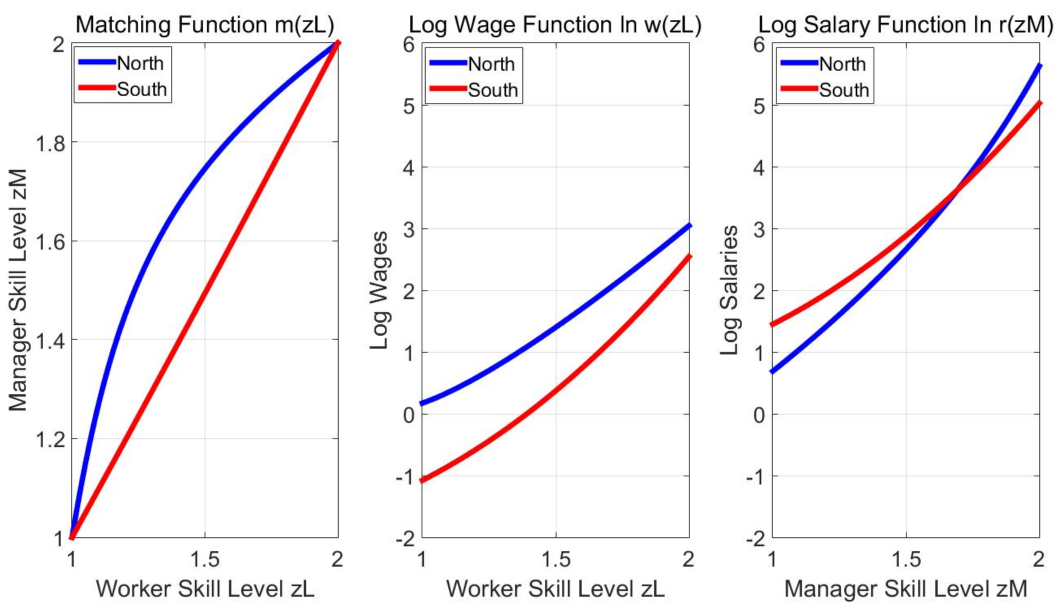
(iii) The size of the most skilled firms (weakly) increases, whereas the size of the least skilled firms (weakly) decreases.

Proof. See Appendix A.9. □

Table 3: Sets of parameter values for Proposition 10

Parameter	Description	Value (North)	Value (South)
$\bar{M}$	Number of managers	100	100
$\bar{L}$	Number of workers	1,000	1,000
$\mathcal{M}$	Set of manager skill levels	[1,2]	[1,2]
$\mathcal{L}$	Set of worker skill levels	[1,2]	[1,2]
k_M	Shape parameter for manager skill distribution	2	2
k_L	Shape parameter for worker skill distribution	2	2
α	Hicks-neutral technology	1	1
β	Sensitivity to manager skill level	1.2	1.2
γ	Span of control	**0.65**	**0.45**

Figure 3: The distributional impact of improvement in management technology



In Table 3, we present sets of parameter values that characterize Propostion 10. Figure 3 shows North and South matching functions, log wage functions, and log salary functions given in Proposition 10.

Contrary to the Hicks-neutral technology transfer case, the ratio of the marginal product of the manager to the marginal product of the worker changes due to the adoption of management technology. This implies that the optimal number of workers given a production unit could be affected, which results in the change of the matching function. Because each manager can accommodate the larger number of workers, workers now match with higher skilled managers. The size of the production unit per manager increases in the high-skilled manager group while the size decreases in the low-skilled manager group. The change in the relative factor endowment will feed through into higher salaries (lower wages) for the high-skilled manager group (the high-skilled worker group) and lower salaries (higher wages) for the low- skilled manager group (the low-skilled worker group). The increase in management technology also can be interpreted as an increase in the marginal product of the worker. In each production unit, the workers share increases and the managers share decreases. As the matching function shifts upward, workers gain while managers lose from the matching effect.

5.3 Manager-Biased Technical Change

Proposition 11. *Suppose that the South can adopt a manager-biased Northern technology - i.e., from $\alpha e^{z_M^{\beta^S} z_L} N^\gamma$ to $\alpha e^{z_M^{\beta^{S'}} z_L} N^\gamma$ such that $\beta^{S'} = \beta^N > \beta^S$. Also, assume that $z^N_{L,min} = z^S_{L,min} \geq 1$ and $z^N_{M,min} = z^S_{M,min} \geq 1$. Then, in the South,*

(i) The matching function shifts upward;

(ii) The wage schedule shifts upward and wage inequality rises;

(iii) The highest skilled manager's salary rises;

(iv) The size of the most skilled firms (weakly) increases, whereas the size of the least skilled firms (weakly) decreases.

Proof. See Appendix A.10. □

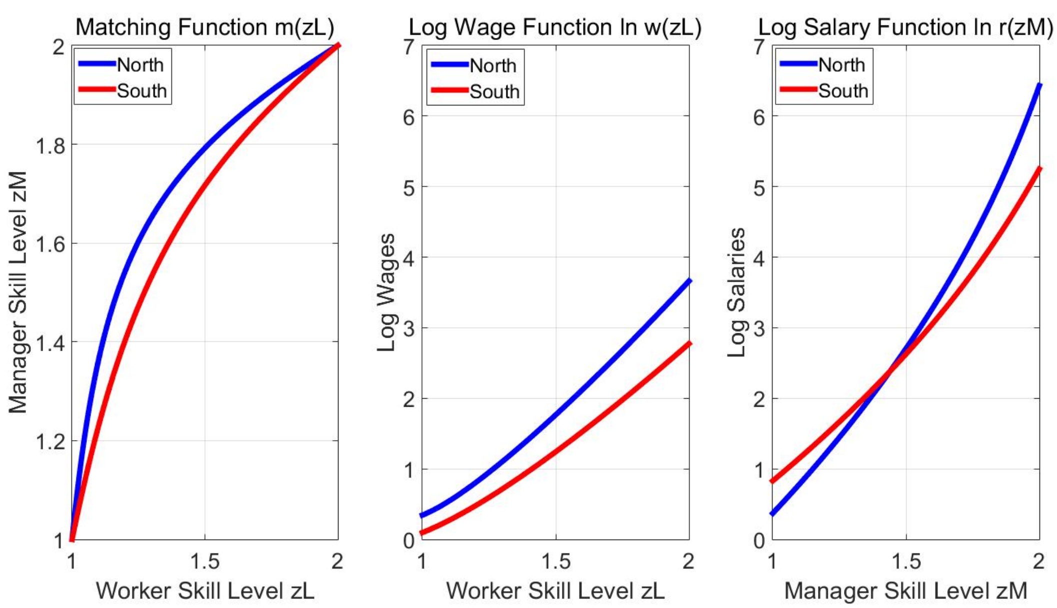
In Table 4, we present sets of parameter values that characterize Propostion 11. Figure 4 shows North and South matching functions, log wage functions, and log salary functions given in Proposition 11.

As in the Hicks-neutral technology transfer case, the ratio of the marginal product of the manager to the marginal product of the worker does not change. However, the elasticity of productivity with respect to worker skill and the elasticity of productivity with respect to manager skill are affected by the manager-biased technical change. This, in turn, affects the trade-off relation between productivity and wage (salary), which changes the matching function. The increase in the parameter β is equivalent to a skill upgrading for managers. This leads to an upward shift in the matching function. The highest-skilled manager can manage more workers, which bids up the salary of the manager through the factor intensity effect. Also, the skill upgrading effect feeds through the salary positively. When these two effects are combined the highest-skilled manager's salary rises. As the matching function and the parameter β increase, wage inequality widens among workers. The minimum wage rises because the size of production units diminishes among low-skilled worker groups, and an increase in the parameter β is equivalent to a partner's skill upgrading.

Table 4: Sets of parameter values for Proposition 11

Parameter	Description	Value (North)	Value (South)
$\bar{M}$	Number of managers	100	100
$\bar{L}$	Number of workers	1,000	1,000
$\mathcal{M}$	Set of manager skill levels	[1,2]	[1,2]
$\mathcal{L}$	Set of worker skill levels	[1,2]	[1,2]
k_M	Shape parameter for manager skill distribution	2	2
k_L	Shape parameter for worker skill distribution	2	2
α	Hicks-neutral technology	1	1
β	Sensitivity to manager skill level	**1.4**	**1.1**
γ	Span of control	0.65	0.65

Figure 4: The distributional impact of manager-biased technical change



6 Endogenizing Task

We study the impacts of offshoring on matching between a manager and workers and earnings structure of the economy when agents are allowed to choose tasks endogenously. The endogenous occupational choice setting enables us to compare ours with the model of Antràs, Garicano and Rossi-Hansberg (2006) where they analyze the formation of cross-country team affects the organization of work, occupational choices, and the structure of earnings. We relax the assumption of particular functional forms for the distribution of ability and the production function in Antràs, Garicano and Rossi-Hansberg (2006) and derive general results regarding the distributionl impacts of offshoring.[26]

6.1 Cross-Country Differences in Factor Distributions (General Case)

Suppose that there are $\bar{L}$ units of inelastic "agents" and they are indexed by only their skill level $z \in \mathcal{Z} \subset \mathbb{R}_{++}$. $\phi(z)$ is a probability density function over skill z which is continuous and strictly positive over their bounded supports, $\mathcal{Z} = [z_{min}, z_{max}]$ where z_{min} and z_{max} denote the lowest skill level of agents and the highest skill level of agents, respectively. $\Phi(z)$ is a cumulative distribution function for agent skill z. We assume that production technology is the same as in the exogenous supplies of managers and workers.

Assumption 1. $\dfrac{\beta m(z)^{\beta-1} z}{1-\gamma} > \dfrac{m(z)^{\beta}}{\gamma}$, for all $z \in \mathcal{Z}$.

Assumption 1, equation (3), and equation (7) determine the equilibrium sorting pattern. There exists a cutoff skill level z_* such that all agents with skill above z_* become managers and all agents with skill below z_* become workers.[27]

Definition 5. (Competitive equilibrium with endogenous task choice) *Under the assumption 1, a competitive equilibrium with endogenous task choice is characterized by a cutoff z_* and a set of functions $m : [z_{min}, z_*] \to [z_*, z_{max}]$, $w : [z_{min}, z_*] \to \mathbb{R}_{++}$, and $r : [z_*, z_{max}] \to \mathbb{R}_{++}$ such that*

i) Cutoff condition:

$$w(z_*) = r(z_*).$$

[26] In Antràs, Garicano and Rossi-Hansberg (2006), they briefly outline the robustness of their key results to more general distributions in the IV.D. Generalizations section. We analyze the general case and provide comparative statics results. We illustrate four possible cases of the general result and find that the result of Antràs, Garicano and Rossi-Hansberg (2006) is one special case of the general case.

[27] Since the profit maximization problem of the endogenous task case is the same as in the exogenous case, we skip the details of derivation.

ii) Differential equations with two boundary conditions:

$$\frac{m(z)^{\beta}}{\gamma} = \frac{w'(z)}{w(z)}, \quad \text{for all } z \in [z_{min}, z_*],$$

$$\frac{\beta z_M^{\beta-1} m^{-1}(z)}{1-\gamma} = \frac{r'(z)}{r(z)}, \quad \text{for all } z \in [z_*, z_{max}],$$

$$m'(z) \left[\frac{\gamma \alpha e^{m(z)^{\beta} z}}{w(z)} \right]^{1/1-\gamma} \phi(m(z)) = \phi(z), \quad \text{for all } z \in [z_{min}, z_*],$$

with $z_* = m(z_{min})$ *and* $z_{max} = m(z_*)$.

Lemma 5. *Suppose* $\frac{\phi^N(z')}{\phi^N(z)} \geq \frac{\phi^S(z')}{\phi^S(z)} \quad \forall z' \geq z$. *Then,* $z_*^N \geq z_*^S$.

Proof. See Appendix A.11. □

Because the relative supply of high skilled agents is more abundant in the North than in the South and more high-skilled agents become managers, the cutoff skill level z_* is higher in the North than in the South. Notably, a middle-skilled agent who is a worker in the North can be a manager in the South.

Proposition 12. *Suppose that the North and the South are identical except that there are relatively more high skilled agents in the North than in the South,* $\frac{\phi^N(z')}{\phi^N(z)} \geq \frac{\phi^S(z')}{\phi^S(z)} \quad \forall z' \geq z$ *allowing for different supports of skill distribution functions, i.e.* $\mathcal{Z}^L \neq \mathcal{Z}^S$ *or* $\mathcal{Z}^L = \mathcal{Z}^S$. *If the economy moves from (complete) autarky to (complete) globalization, then*

(i) $z_*^N \geq z_*^W \geq z_*^S$.

(ii) In the North, there exists skill ψ *such that*

(ii-1) Workers become managers, $\quad \forall z \in [z_*^W, z_*^N]$*;*

(ii-2) $m^{N^{-1}}(z) \geq m^{W^{-1}}(z) \quad \forall z \in [\psi, z_{N,max}]$.

(iii) In the South, there exists a skill ζ *such that*

(iii-1) $m^S(z) \leq m^W(z) \quad \forall z \in [z_{S,min}, \zeta]$*;*

(iii-2) Managers become workers, $\quad \forall z \in [z_*^S, z_*^W]$.

(iv) In the North, the inequality widens for $z \in [z_*^W, z_*^N]$ *and the inequality narrows for* $z \in [\psi, z_{N,max}]$.

(v) In the South, the inequality widens for $z \in [z_{S,min}, \zeta]$ *and the inequality narrows for* $z \in [z_*^S, z_*^W]$.

Proof. See Appendix A.12. □

Unlike previous cases where offshoring has no impact on occupational choices, the frictionless cross-country matching changes task decisions for some agents in the North and the South. In the North, some of the most skilled workers become managers. This occupation re-sorting implies that the highly skilled managers in the North whose skill levels $z \in [\psi, z_{N,max}]$ now match with lesser skilled workers from offshoring. The task changes from workers to managers are beneficial to low-skilled and high-skilled agents, but the high-skilled agents benefit more because of the complementary effect between manager skill and worker skill, which implies that within-group inequality widens among agents who change their tasks. For the most high-skilled managers in the North, the downgrading of the matching partner's skill reduces salary inequality.

In the South, some of the least skilled managers become workers. This occupation re-sorting implies that the least skilled workers in the South whose skill levels $z \in [z_{min}, \zeta]$ now match with more skilled managers from offshoring. The task changes from managers to workers reduce within-group inequality. For the least skilled workers in the South, the upgrading of the matching partner's skill increases wage inequality.

While our result (Proposition 12-(i)) re-confirms the key insight from Antràs, Garicano and Rossi-Hansberg (2006) such that globalization leads to the creation of worker jobs in the South and their destruction in the North, our findings on matching patterns and their distributional impacts differ from theirs. We divide four possible cases as follows: Special Case I - $\mathcal{Z}^N = \mathcal{Z}^S = [z_{min}, z_{max}]$, Special Case II - $\mathcal{Z}^N = [z_{N,min}, z_{max}]$ and $\mathcal{Z}^S = [z_{S,min}, z_{max}]$ with $z_{N,min} > z_{S,min}$, Special Case III - $\mathcal{Z}^N = [z_{min}, z_{N,max}]$ and $\mathcal{Z}^S = [z_{min}, z_{S,max}]$ with $z_{N,max} > z_{S,max}$, and Special Case IV - $m^S(z_*^S) = m^N(z_{N,min}) = z_*^N$. It turns out that the result of Antràs, Garicano and Rossi-Hansberg (2006) is one of the four possible cases (Special Case III). Among the four possible cases, we find the Special Case IV especially interesting as globalization can lead to an increase in within-worker inequality in both countries.

6.1.1 Special Case I: $\mathcal{Z}^N = \mathcal{Z}^S = [z_{min}, z_{max}]$

Proposition 13. *Suppose that the North and the South are identical including the same supports of probability distribution functions $\mathcal{Z}^N = \mathcal{Z}^S = [z_{min}, z_{max}]$, except that there are relatively more high skilled agents in the North than in the South, $\dfrac{\phi^N(z')}{\phi^N(z)} \geq \dfrac{\phi^S(z')}{\phi^S(z)} \quad \forall z' \geq z$. If the economy moves from (complete) autarky to (complete) globalization, then*

(i) $z_*^N \geq z_*^W \geq z_*^S$.

(ii) In the North, there exists a cutoff skill ψ such that

(ii-1) $m^N(z) \geq m^W(z) \quad \forall z \in [z_{min}, \psi]$ and $m^N(z) \leq m^W(z) \quad \forall z \in [\psi, z_*^W]$;

(ii-2) Workers become managers, $\forall z \in [z_*^W, z_*^N]$*;*

(ii-3) $m^{N^{-1}}(z) \leq m^{W^{-1}}(z) \quad \forall z \in [z_*^N, m^W(\psi)]$ *and* $m^{N^{-1}}(z) \geq m^{W^{-1}}(z) \quad \forall z \in [m^W(\psi), z_{max}]$*.*

(iii) In the South, there exists a cutoff skill ζ *such that*

(iii-1) $m^S(z) \leq m^W(z) \quad \forall z \in [z_{min}, \zeta]$ *and* $m^S(z) \geq m^W(z) \quad \forall z \in [\zeta, z_*^S]$*;*

(iii-2) Managers become workers, $\forall z \in [z_*^S, z_*^W]$*;*

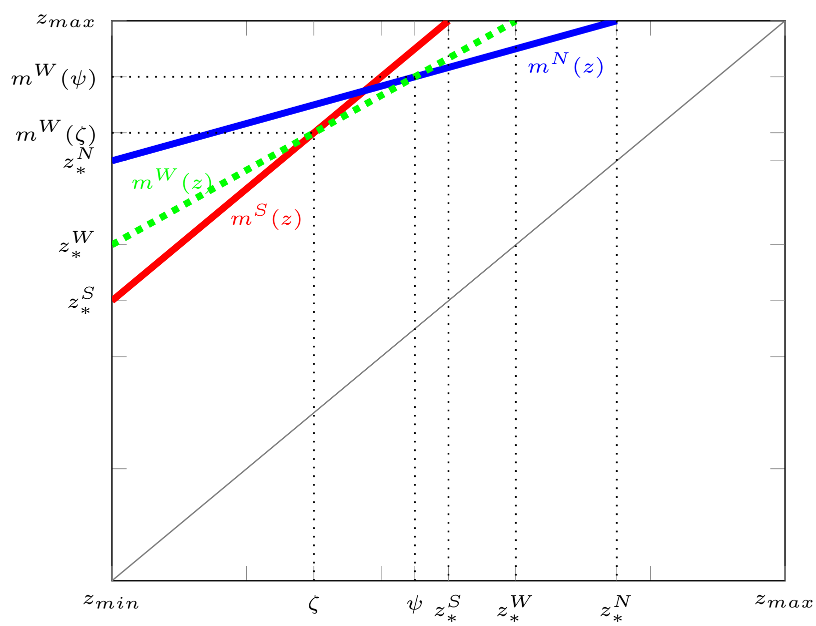
(iii-3) $m^{S^{-1}}(z) \geq m^{W^{-1}}(z) \quad \forall z \in [z_*^W, m^W(\zeta)]$ *and* $m^{S^{-1}}(z) \leq m^{W^{-1}}(z) \quad \forall z \in [m^W(\zeta), z_{max}]$*.*

(iv) In the North, the inequality narrows for $z \in [z_{min}, \psi]$*, the inequality widens for* $z \in [\psi, m^W(\psi)]$*, and the inequality narrows for* $z \in [m^W(\psi), z_{max}]$*.*

(v) In the South, the inequality widens for $z \in [z_{min}, \zeta]$*, the inequality narrows for* $z \in [\zeta, m^W(\zeta)]$*, and the inequality widens for* $z \in [m^W(\zeta), z_{max}]$*.*

Proof. See Appendix A.13. □

Figure 5: Matching functions in the special case I



In the special case I, the supports of skill distributions are identical between two countries. Figure 5 presents the matching functions before and after globalization. For both countries, the old matching function crosses the new matching function. However, the Northern matching function crosses the world matching function from above while the Souther matching function crosses the world matching from below. Hence, the inequality implications are exactly the opposite for both countries.

6.1.2 Special Case II: $\mathcal{Z}^N = [z_{N,min}, z_{max}]$ and $\mathcal{Z}^S = [z_{S,min}, z_{max}]$ with $z_{N,min} > z_{S,min}$

Proposition 14. *Suppose that the North and the South are identical, except that there are relatively more high skilled agents in the North than in the South, $\dfrac{\phi^N(z')}{\phi^N(z)} \geq \dfrac{\phi^S(z')}{\phi^S(z)} \quad \forall z' \geq z$ and $\mathcal{Z}^N = [z_{N,min}, z_{max}]$ and $\mathcal{Z}^S = [z_{S,min}, z_{max}]$ with $z_{N,min} > z_{S,min}$. If the economy moves from (complete) autarky to (complete) globalization, then*

(i) $z^N_* \geq z^W_* \geq z^S_*$.

(ii) In the North, there exists skill ψ such that

(ii-1) Workers become managers, $\quad \forall z \in [z^W_*, z^N_*]$;

(ii-2) $m^{N^{-1}}(z) \geq m^{W^{-1}}(z) \quad \forall z \in [\psi, z_{N,max}]$.

(iii) In the South, there exists a cutoff skill ζ such that

(iii-1) $m^S(z) \leq m^W(z) \quad \forall z \in [z_{min}, \zeta]$ *and* $m^S(z) \geq m^W(z) \quad \forall z \in [\zeta, z^S_*]$;

(iii-2) Managers become workers, $\quad \forall z \in [z^S_*, z^W_*]$;

(iii-3) $m^{S^{-1}}(z) \geq m^{W^{-1}}(z) \quad \forall z \in [z^W_*, m^W(\zeta)]$ *and* $m^{S^{-1}}(z) \leq m^{W^{-1}}(z) \quad \forall z \in [m^W(\zeta), z_{max}]$.

(iv) In the North, the inequality widens for $z \in [z^W_*, z^N_*]$ and the inequality narrows for $z \in [\psi, z_{max}]$.

(v) In the South, the inequality widens for $z \in [z_{min}, \zeta]$, the inequality narrows for $z \in [\zeta, m^W(\zeta)]$, and the inequality widens for $z \in [m^W(\zeta), z_{max}]$.

Proof. See Proof of Proposition 13. □

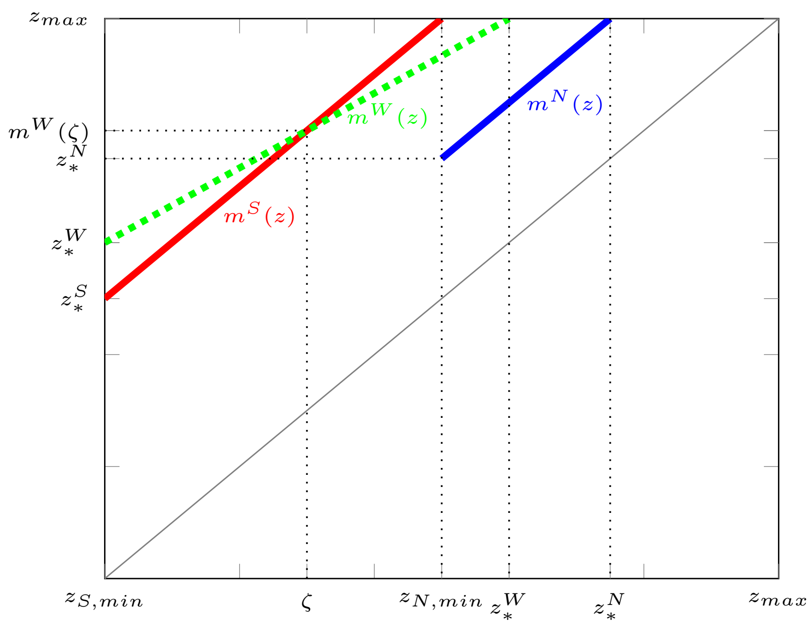
In the special case II, the skill level of the least skilled agent in the South is strictly lower than the one of the least skilled agent in the North while the skill levels of most skilled agents in both countries are the same. Figure 6 presents the matching functions before and after globalization. In the South, the old matching function crosses the world matching from below. However, in the North, the matching function shifts to the left. Hence, within-worker inequality rises while within-manager inequality reduces in the North.

6.1.3 Special Case III: $\mathcal{Z}^N = [z_{min}, z_{N,max}]$ and $\mathcal{Z}^S = [z_{min}, z_{S,max}]$ with $z_{N,max} > z_{S,max}$

Proposition 15. *Suppose that the North and the South are identical, except that there are relatively more high skilled agents in the North than in the South, $\dfrac{\phi^N(z')}{\phi^N(z)} \geq \dfrac{\phi^S(z')}{\phi^S(z)} \quad \forall z' \geq z$ and $\mathcal{Z}^N = [z_{min}, z_{N,max}]$ and $\mathcal{Z}^S = [z_{min}, z_{S,max}]$ with $z_{N,max} > z_{S,max}$. If the economy moves from (complete) autarky to (complete) globalization, then*

(i) $z^N_* \geq z^W_* \geq z^S_*$.

Figure 6: Matching functions in the special case II



(ii) In the North, there exists a cutoff skill ψ such that

(ii-1) $m^N(z) \geq m^W(z) \quad \forall z \in [z_{min}, \psi]$ and $m^N(z) \leq m^W(z) \quad \forall z \in [\psi, z_*^W]$;

(ii-2) Workers become managers, $\quad \forall z \in [z_*^W, z_*^N]$;

(ii-3) $m^{N^{-1}}(z) \leq m^{W^{-1}}(z) \quad \forall z \in [z_*^N, m^W(\psi)]$ and $m^{N^{-1}}(z) \geq m^{W^{-1}}(z) \quad \forall z \in [m^W(\psi), z_{max}]$.

(iii) In the South, there exists a skill ζ such that

(iii-1) $m^S(z) \leq m^W(z) \quad \forall z \in [z_{S,min}, \zeta]$;

(iii-2) Managers become workers, $\quad \forall z \in [z_*^S, z_*^W]$.

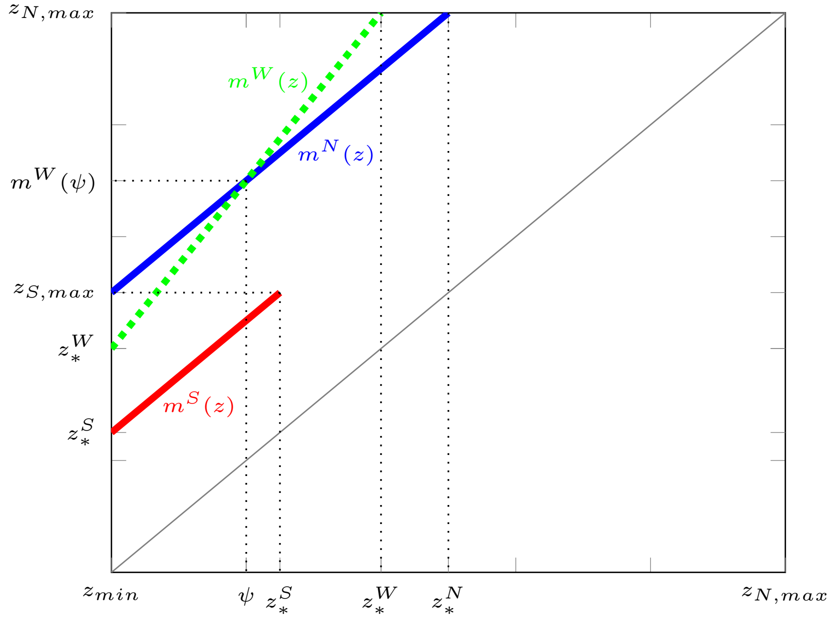
(iv) In the North, the inequality narrows for $z \in [z_{min}, \psi]$, the inequality widens for $z \in [\psi, m^W(\psi)]$, and the inequality narrows for $z \in [m^W(\psi), z_{max}]$.

(v) In the South, the inequality widens for $z \in [z_{min}, \zeta]$ and the inequality narrows for $z \in [z_*^S, z_*^W]$.

Proof. See Proof of Proposition 13. □

In the special case III, the skill level of the most skilled agent in the North is strictly larger than the one of the most skilled agent in the South while the skill levels of least skilled agents in both countries are the same. Figure 7 presents the matching functions before and after globalization. In the North, the old matching function crosses the world matching from above. However, in the South, the matching function shifts upward. Hence, within-worker inequality rises while within-manager inequality narrows in the

Figure 7: Matching functions in the special case III



South. The inequality implication in this case is identical to that of Antràs, Garicano and Rossi-Hansberg (2006).

6.1.4 Special Case IV: $m^S(z_*^S) = m^N(z_{N,min}) = z_*^N$

Proposition 16. *Suppose that the North and the South are identical, except that there are relatively more high skilled agents in the North than in the South,* $\dfrac{\phi^N(z')}{\phi^N(z)} \geq \dfrac{\phi^S(z')}{\phi^S(z)} \quad \forall z' \geq z$ *and* $m^S(z_*^S) = m^N(z_{N,min}) = z_*^N$*. If the economy moves from (complete) autarky to (complete) globalization, then*

(i) $z_*^N \geq z_*^W \geq z_*^S$.

(ii) In the North,

(ii-1) $m^N(z) \leq m^W(z) \quad \forall z \in [z_{N,min}, z_*^W]$;

(ii-2) Workers become managers, $\quad \forall z \in [z_*^W, z_*^N]$;

(ii-3) $m^{N^{-1}}(z) \geq m^{W^{-1}}(z) \quad \forall z \in [z_*^N, z_{N,max}]$.

(iii) In the South,

(iii-1) $m^S(z) \leq m^W(z) \quad \forall z \in [z_{S,min}, z_*^S]$;

(iii-2) Managers become workers, $\quad \forall z \in [z_*^S, z_*^W]$;

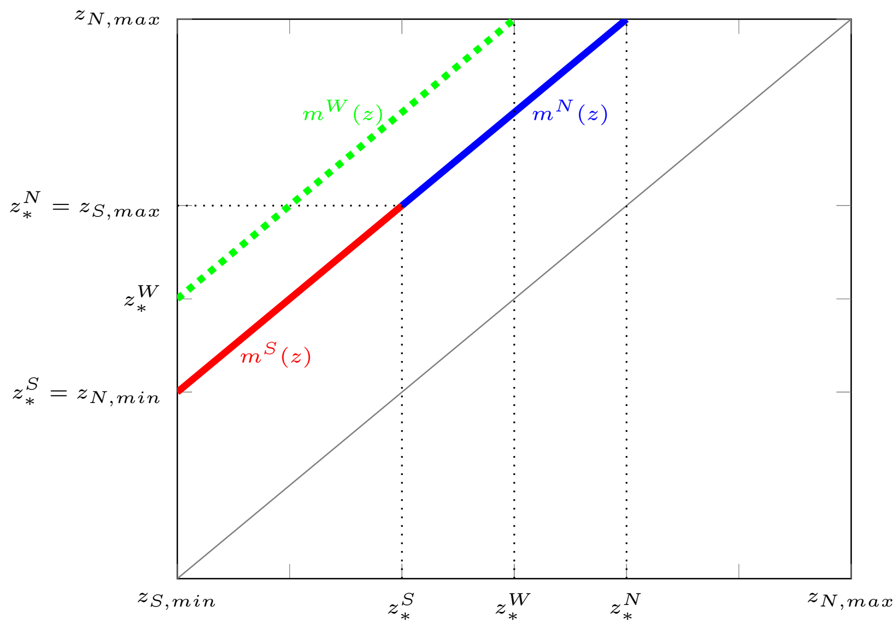
(iii-3) $m^{S^{-1}}(z) \geq m^{W^{-1}}(z) \quad \forall z \in [z_*^W, z_{S,max}]$.

(iv) In the North, the inequality widens for $z \in [z_{N,min}, z_*^N]$ *and the inequality narrows for* $z \in [z_*^N, z_{N,max}]$.

(v) In the South, the inequality widens for $z \in [z_{S,min}, z_*^S]$ and the inequality narrows for $z \in [z_*^S, z_{S,max}]$.

Proof. See Appendix A.14. □

Figure 8: Matching functions in the special case IV



In the special case IV, the skill level of the most (least) skilled agent in the North is strictly larger than the one of the most (least) skilled agent in the South. Figure 8 presents the matching functions before and after globalization. Interestingly, the matching functions in both countries shift upward, implying that the within-worker inequality in both countries rises while the within-manager inequality in both countries shrinks from offshoring.

7 Two-Industry Model

In this section, we extend the model to allow for two industries, which is the same set-up as in Grossman, Helpman and Kircher (2017). They study how relative output prices affect sorting and matching patterns that lead to distributional consequences. We study a case in which relative output prices are identical between two countries, while factor distributions differ across two countries. Because the relative output prices are the same, there would be no international trade between two countries from Grossman, Helpman and Kircher (2017)'s perspective. Then, can offshoring affect the matching patterns and the distribution of earnings even when the relative output prices are identcal? In the following analysis, we illustrate the mechanism of international trade and offshoring are clearly different.

The production function in industry i is defined as follows:

$$Y_i = F_i(z_M, z_L, N) = \alpha e^{z_M^{\beta_i} z_L} N^{\gamma_i}, \quad \alpha \in \{0,1\}, \quad \beta_i > 1, \quad 0 < \gamma_i < 1, \text{ for } i = 1, 2.$$

Assumption 2. $\dfrac{z_M^{\beta_1}}{\gamma_1} > \dfrac{z_M^{\beta_2}}{\gamma_2}$ and $\dfrac{\beta_1 z_M^{\beta_1 - 1} z_L}{1-\gamma_1} > \dfrac{\beta_2 z_M^{\beta_2 - 1} z_L}{1-\gamma_2}$, for all $z_M \in \mathcal{M}, z_L \in \mathcal{L}$.

Assumption 2 guarantees that the more skilled managers with $z_M > z_M^*$ and the more skilled workers with $z_L > z_L^*$ are employed in sector 1, while the less skilled managers with $z_M < z_M^*$ and the less skilled workers with $z_L < z_L^*$ are employed in sector 2, for some $z_M^* \in \mathcal{M}$ and some $z_L^* \in \mathcal{L}$. We define p_1 as the price of good 1 and p_2 as the price of good 2.

Definition 6. (Competitive equilibrium with two industries) *Under the assumption 2, a competitive equilibrium with two industries is characterized by cutoffs $\{z_M^*, z_L^*\}$ and a set of functions $m_1 : [z_L^*, z_{L,max}] \to [z_M^*, z_{M,max}]$, $m_2 : [z_{L,min}, z_L^*] \to [z_{M,min}, z_M^*]$, $w_1 : [z_L^*, z_{L,max}] \to \mathbb{R}_{++}$, $w_2 : [z_{L,min}, z_L^*] \to \mathbb{R}_{++}$, $r_1 : [z_M^*, z_{M,max}] \to \mathbb{R}_{++}$, and $r_2 : [z_{M,min}, z_M^*] \to \mathbb{R}_{++}$ such that*

i) Cutoff condition:

$$w_1(z_L^*) = w_2(z_L^*),$$

$$r_1(z_M^*) = r_2(z_M^*).$$

ii) Industry 1 differential equations with two boundary conditions:

$$\frac{m_1(z_L)^{\beta_1}}{\gamma_1} = \frac{w_1'(z_L)}{w_1(z_L)}, \quad \text{for all } z_L \in [z_L^*, z_{L,max}],$$

$$\frac{\beta_1 z_M^{\beta_1 - 1} m_1^{-1}(z_M)}{1-\gamma_1} = \frac{r_1'(z_M)}{r_1(z_M)}, \quad \text{for all } z_M \in [z_M^*, z_{M,max}],$$

$$\bar{M} m_1'(z_L) \left[\frac{\gamma_1 \alpha p_1 e^{m_1(z_L)^{\beta_1} z_L}}{w_1(z_L)} \right]^{1/1-\gamma_1} \phi_M(m_1(z_L)) = \bar{L} \phi_L(z_L), \quad \text{for all } z_L \in [z_L^*, z_{L,max}],$$

with $z_M^* = m_1(z_L^*)$ and $z_{M,max} = m_1(z_{L,max})$.

iii) Industry 2 differential equations with two boundary conditions:

$$\frac{m_2(z_L)^{\beta_2}}{\gamma_2} = \frac{w_2'(z_L)}{w_2(z_L)}, \quad \text{for all } z_L \in [z_{L,min}, z_L^*],$$

$$\frac{\beta_2 z_M^{\beta_2-1} m_2^{-1}(z_M)}{1-\gamma_2} = \frac{r_2'(z_M)}{r_2(z_M)}, \quad \text{for all } z_M \in [z_{M,min}, z_M^*],$$

$$\bar{M} m_2'(z_L) \left[\frac{\gamma_2 \alpha p_2 e^{m_2(z_L)^{\beta_2} z_L}}{w_2(z_L)} \right]^{1/1-\gamma_2} \phi_M(m_2(z_L)) = \bar{L}\phi_L(z_L), \quad \text{for all } z_L \in [z_{L,min}, z_L^*],$$

with $z_{M,min} = m_2(z_{L,min})$ *and* $z_M^* = m_2(z_L^*)$.

Example 1. *Suppose that the North and the South are identical including relative output prices* $(\frac{p_1}{p_2})^N = (\frac{p_1}{p_2})^S$, *except that there are relatively more high skilled workers in the North than in the South,* $\frac{\phi_L^N(z_L')}{\phi_L^N(z_L)} > \frac{\phi_L^S(z_L')}{\phi_L^S(z_L)}$ $\forall z_L' > z_L$. *Then, matching functions in both countries are not the same. Thus, there does not exist international trade between the two countries because relative output prices are identical, while there exist offshoring between the two countries if the economy moves from (complete) autarky to (complete) globalization.*

Proof. See Appendix A.15 □

7.1 Two-Industry Model with Endogenous Task Choice

Assumption 3. $\frac{\beta_1 m(z)^{\beta_1-1} z}{1-\gamma_1} > \frac{\beta_2 m(z)^{\beta_2-1} z}{1-\gamma_2} > \frac{m(z)^{\beta_1}}{\gamma_1} > \frac{m(z)^{\beta_2}}{\gamma_2}$, for all $z \in \mathcal{Z}$.

Assumption 3 determines the equilibrium sorting pattern. There exist three cutoff skill levels $\{\widetilde{z}, \overline{z}, z^*\}$ such that all agents with skill level $[\widetilde{z}, z_{max}]$ are employed in sector 1 as managers, all agents with skill level $[\overline{z}, \widetilde{z}]$ are employed in sector 2 as managers, all agents with skill level $[z^*, \overline{z}]$ are employed in sector 1 as workers, and all agents with skill level $[z_{min}, z^*]$ are employed in sector 2 as workers.

Definition 7. (Competitive equilibrium with two industries and endogenous task choice) *Under the assumption 3, a competitive equilibrium with two industries and endogenous task choice is characterized by cutoffs* $\{\widetilde{z}, \overline{z}, z^*\}$ *and a set of functions* $m_1 : [z^*, \overline{z}] \to [\widetilde{z}, z_{max}]$, $m_2 : [z_{min}, z^*] \to [\overline{z}, \widetilde{z}]$, $w_1 : [z^*, \overline{z}] \to \mathbb{R}_{++}$, $w_2 : [z_{min}, z^*] \to \mathbb{R}_{++}$, $r_1 : [\widetilde{z}, z_{max}] \to \mathbb{R}_{++}$, *and* $r_2 : [\overline{z}, \widetilde{z}] \to \mathbb{R}_{++}$ *such that*

i) Cutoff condition:

$$w_1(z^*) = w_2(z^*),$$

$$w_2(\overline{z}) = r_1(\overline{z}),$$

$$r_1(\widetilde{z}) = r_2(\widetilde{z}).$$

ii) Industry 1 differential equations with two boundary conditions:

$$\frac{m_1(z)^{\beta_1}}{\gamma_1} = \frac{w_1'(z)}{w_1(z)}, \quad \text{for all } z \in [z^*, \overline{z}],$$

$$\frac{\beta_1 z^{\beta_1 - 1} m_1^{-1}(z)}{1-\gamma_1} = \frac{r_1'(z)}{r_1(z)}, \quad \text{for all } z \in [\widetilde{z}, z_{max}],$$

$$\bar{M} m_1'(z) \left[\frac{\gamma_1 \alpha p_1 e^{m_1(z)^{\beta_1} z}}{w_1(z)} \right]^{1/1-\gamma_1} \phi(m_1(z)) = \bar{L}\phi(z), \quad \text{for all } z \in [z^*, \overline{z}],$$

with $\widetilde{z} = m_1(z^*)$ *and* $z_{max} = m_1(\overline{z})$.

iii) Industry 2 differential equations with two boundary conditions:

$$\frac{m_2(z)^{\beta_2}}{\gamma_2} = \frac{w_2'(z)}{w_2(z)}, \quad \text{for all } z \in [z_{min}, z^*],$$

$$\frac{\beta_2 z^{\beta_2 - 1} m_2^{-1}(z)}{1-\gamma_2} = \frac{r_2'(z)}{r_2(z)}, \quad \text{for all } z \in [\overline{z}, \widetilde{z}],$$

$$\bar{M} m_2'(z) \left[\frac{\gamma_2 \alpha p_2 e^{m_2(z)^{\beta_2} z}}{w_2(z)} \right]^{1/1-\gamma_2} \phi(m_2(z)) = \bar{L}\phi(z), \quad \text{for all } z \in [z_{min}, z^*],$$

with $\overline{z} = m_2(z_{min})$ *and* $\widetilde{z} = m_2(z^*)$.

Example 2. *Suppose that the North and the South are identical including relative output prices* $(\frac{p_1}{p_2})^N = (\frac{p_1}{p_2})^S$*, except that there are relatively more high skilled agents in the North than in the South,* $\frac{\phi^N(z')}{\phi^N(z)} > \frac{\phi^S(z')}{\phi^S(z)} \quad \forall z' > z$*. Then, matching functions in both countries are not the same. Thus, there does not exist international trade between the two countries because relative output prices are identical, while there exist offshoring between the two countries if the economy moves from (complete) autarky to (complete) globalization.*

Proof. See Appendix A.16 □

8 Conclusion

This paper develops a matching framework of offshoring where offshoring is defined as a cross-country matching between a manager and workers. Our model features two countries and two factors of production with heterogeneous skills in perfect competition. Most importantly, production technology is characterized by complementarity between workers and managers. We have analyzed the effects of offshoring on the matching patterns and the structure of earnings of heterogeneous individuals when two countries are different in aspects such as factor endowments, factor distributions, and technology levels. Then, we relax the assumption of exogenous supplies of agents and endogenize the occupational choice as in Antràs, Garicano and Rossi-Hansberg (2006), and derive general results of cross-country team formation on matching patterns and occupational decisions that encompass the result of Antràs, Garicano and Rossi-Hansberg (2006). In one particular case, we obtain a counterintuitive result such that all Northern workers and Southern workers match with better managers from offshoring, and thus offshoring increases within-worker inequality in both countries. Finally, we allow for two industries and compare our mechanism of offshoring with the international trade mechanism in Grossman, Helpman and Kircher (2017)'s matching model. We show that offshoring can happen even when relative output prices are identical between two countries, which illustrates that the international trade and the offshoring are not identical.

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Appendix

A Proofs

A.1 Proof of Proposition 1

Proof. Let $\pi(z_M, z_L)$ denote the profit of a firm hiring a manager of skill z_M and employing the optimal number of workers $L(z_L; z_M)$ of skill z_L. Plugging the conditional worker demand in equation (2) into (1) yields,

$$\begin{aligned}\pi(z_M, z_L) &= \alpha e^{z_M^\beta z_L} \left[\frac{\gamma \alpha e^{z_M^\beta z_L}}{w(z_L)}\right]^{\gamma/(1-\gamma)} - w(z_L) \left[\frac{\gamma \alpha e^{z_M^\beta z_L}}{w(z_L)}\right]^{1/(1-\gamma)} \\ &= \gamma^{\gamma/(1-\gamma)}(1-\gamma)\alpha^{1/(1-\gamma)} \left[e^{z_M^\beta z_L}\right]^{1/(1-\gamma)} w(z_L)^{-\gamma/1-\gamma}.\end{aligned}$$

In equilibrium, firms choose workers' skill level z_L to maximize profits:

$$\frac{\partial \pi(z_M, z_L)}{\partial z_L} = \gamma^{\gamma/(1-\gamma)}\alpha^{1/(1-\gamma)} \left[e^{z_M^\beta z_L}\right]^{1/(1-\gamma)} \left[z_M^\beta w(z_L)^{-\gamma/1-\gamma} - \gamma w(z_L)^{-1/1-\gamma} w'(z_L)\right] = 0. \tag{14}$$

Totally differentiating the expression yields,

$$\frac{\partial z_M}{\partial z_L} = -\frac{\partial^2 \pi(z_M, z_L)/\partial z_L^2}{\partial^2 \pi(z_M, z_L)/\partial z_L \partial z_M}.$$

Given that firms maximize profits in equilibrium, the numerator must be negative. To show that $m(z_L)$ is an increasing function for $z_L \in \mathcal{L}$, the denominator must be positive.

$$\begin{aligned}\frac{\partial^2 \pi(z_M, z_L)}{\partial z_L \partial z_M} &= \gamma^{\gamma/(1-\gamma)}\alpha^{1/(1-\gamma)} \frac{1}{1-\gamma} \left[e^{z_M^\beta z_L}\right]^{1/1-\gamma} \beta z_M^{\beta-1} z_L \left[z_M^\beta w(z_L)^{-\gamma/1-\gamma} - \gamma w(z_L)^{-1/1-\gamma} w'(z_L)\right] \\ &+ \gamma^{\gamma/(1-\gamma)}\alpha^{1/(1-\gamma)} \left[e^{z_M^\beta z_L}\right]^{1/1-\gamma} \beta z_M^{\beta-1} w(z_L)^{-\gamma/1-\gamma} \\ &= \left[\gamma^{\gamma/1-\gamma}\alpha^{1/1-\gamma} \left[e^{z_M^\beta z_L}\right]^{1/1-\gamma} \beta z_M^{\beta-1} w(z_L)^{-\gamma/1-\gamma}\right].\end{aligned}$$

where the last equality follows from equation (14). To show that the denominator is positive, we must show that $w(z_L)$ is strictly positive. From the profit maximization problem, we know that the (optimal) conditional worker demand $N(z_L; z_M)$ is strictly positive conditional on $\alpha > 0$. This implies that wage schedule $w(z_L)$ is positive. Therefore, in equilibrium, the matching function $m(z_L)$ is a strictly increasing function for $z_L \in \mathcal{L}$. $\square$

A.2 Proof of Proposition 2

Proof. To show that the log wage schedule $\ln w(z_L)$ is strictly increasing, differentiate $\ln w(z_L)$ with respect to z_L:

$$\frac{d \ln w(z_L)}{dz_L} = \frac{m(z_L)^\beta}{\gamma} > 0.$$

Next, to prove that the log wage schedule $\ln w(z_L)$ is convex in skills, differentiate $\frac{d \ln w(z_L)}{dz_L}$ with respect to z_L:

$$\frac{d^2 \ln w(z_L)}{dz_L^2} = \frac{\beta m(z_L)^{\beta-1} m'(z_L)}{\gamma} > 0$$

where the last inequality follows from the positive assortative matching property of the matching function $m(z_L)$. □

A.3 Proof of Proposition 3

Proof. To show that the log salary schedule $\ln r(z_M)$ is strictly increasing, differentiate $\ln r(z_M)$ with respect to z_M:

$$\frac{d \ln r(z_M)}{dz_M} = \frac{\beta z_M^{\beta-1} m^{-1}(z_M)}{1-\gamma} > 0.$$

Next, to prove that the log salary schedule $\ln r(z_M)$ is convex in skills, differentiate $\frac{d \ln r(z_M)}{dz_M}$ with respect to z_M:

$$\frac{d^2 \ln r(z_M)}{dz_M^2} = \frac{\beta(\beta-1) z_M^{\beta-2} m^{-1}(z_M)}{1-\gamma} + \frac{\beta z_M^{\beta-1} m^{-1\prime}(z_M)}{1-\gamma} > 0$$

where the last inequality follows from the positive assortative matching property of the matching function $m(z_L)$ and $\beta > 1$. □

A.4 Proof of Proposition 4

Proof. (i) In Autarky, total production in the World is defined as:

$$\int_{z_{M,min}}^{z_{M,max}} e^{z_M^\beta m^{-1}(z_M)} (N^N)^\gamma \bar{M}^N \phi_M(z_M) dz_M + \int_{z_{M,min}}^{z_{M,max}} e^{z_M^\beta m^{-1}(z_M)} (N^S)^\gamma \bar{M}^S \phi_M(z_M) dz_M \quad (15)$$

where $N^N = \left[\frac{\gamma e^{z_M^\beta m^{-1}(z_M)}}{w^N(z_L)}\right]^{1/1-\gamma}$ and $N^S = \left[\frac{\gamma e^{z_M^\beta m^{-1}(z_M)}}{w^S(z_L)}\right]^{1/1-\gamma}$.

In Globalization, total production in the World is defined as:

$$\int_{z_{M,min}}^{z_{M,max}} e^{z_M^\beta m^{-1}(z_M)} (N^W)^\gamma (\bar{M}^N + \bar{M}^S) \phi_M(z_M) dz_M \tag{16}$$

where $N^W = \left[\frac{\gamma e^{z_M^\beta m^{-1}(z_M)}}{w^W(z_L)} \right]^{1/1-\gamma}$. Using the market clearing condition in equation (8), the conditional worker demand N can be represented as:

$$N = \frac{\bar{L}}{\bar{M}} \frac{\phi_L(m^{-1}(z_M))}{\phi_M(z_M)} \frac{1}{m'(m^{-1}(z_M))}.$$

Plugging this conditional worker demand into equations (15) and (16), we can obtain total production in Autarky and in Globalization, respectively, as follows:

$$\left[(\bar{M}^N)^{1-\gamma} (\bar{L}^N)^\gamma + (\bar{M}^S)^{1-\gamma} (\bar{L}^S)^\gamma \right] \int_{z_{M,min}}^{z_{M,max}} e^{z_M^\beta m^{-1}(z_M)} \left(\frac{\phi_L(m^{-1}(z_M))}{\phi_M(z_M)} \frac{1}{m'(m^{-1}(z_M))} \right)^\gamma \phi_M(z_M) dz_M,$$

$$(\bar{M}^N + \bar{M}^S)^{1-\gamma} (\bar{L}^N + \bar{L}^S)^\gamma \int_{z_{M,min}}^{z_{M,max}} e^{z_M^\beta m^{-1}(z_M)} \left(\frac{\phi_L(m^{-1}(z_M))}{\phi_M(z_M)} \frac{1}{m'(m^{-1}(z_M))} \right)^\gamma \phi_M(z_M) dz_M.$$

Using Theorem 9.4 **(Generalized Hölder's inequality)** in Cvetkovski (2012), for any $\gamma \in (0,1)$ and positive real numbers $\bar{M}^N$, $\bar{M}^S$, $\bar{L}^N$, and $\bar{L}^S$, the following inequality holds:

$$(\bar{M}^N + \bar{M}^S)^{1-\gamma} (\bar{L}^N + \bar{L}^S)^\gamma \geq \left[(\bar{M}^N)^{1-\gamma} (\bar{L}^N)^\gamma + (\bar{M}^S)^{1-\gamma} (\bar{L}^S)^\gamma \right].$$

Equality occurs if and only if $\frac{\bar{M}^N}{\bar{L}^N} = \frac{\bar{M}^S}{\bar{L}^S}$. Therefore, total production in the World strictly increases from globalization.

(ii) In Autakry, total earnings in the North is defined as:

$$(\bar{M}^N)^{1-\gamma} (\bar{L}^N)^\gamma \int_{z_{M,min}}^{z_{M,max}} e^{z_M^\beta m^{-1}(z_M)} \left(\frac{\phi_L(m^{-1}(z_M))}{\phi_M(z_M)} \frac{1}{m'(m^{-1}(z_M))} \right)^\gamma \phi_M(z_M) dz_M.$$

In Globalization, total earnings in the North is defined as:

$$\begin{aligned}
&\frac{\bar{M}^N}{\bar{M}^N+\bar{M}^S}(1-\gamma)(\bar{M}^N+\bar{M}^S)^{1-\gamma}(\bar{L}^N+\bar{L}^S)^{\gamma} \\
&\times \int_{z_{M,min}}^{z_{M,max}} e^{z_M^\beta m^{-1}(z_M)} \left(\frac{\phi_L(m^{-1}(z_M))}{\phi_M(z_M)}\frac{1}{m'(m^{-1}(z_M))}\right)^{\gamma}\phi_M(z_M)dz_M \\
&+\frac{\bar{L}^N}{\bar{L}^N+\bar{L}^S}\gamma(\bar{M}^N+\bar{M}^S)^{1-\gamma}(\bar{L}^N+\bar{L}^S)^{\gamma} \\
&\times \int_{z_{M,min}}^{z_{M,max}} e^{z_M^\beta m^{-1}(z_M)} \left(\frac{\phi_L(m^{-1}(z_M))}{\phi_M(z_M)}\frac{1}{m'(m^{-1}(z_M))}\right)^{\gamma}\phi_M(z_M)dz_M \\
&=(\bar{M}^N+\bar{M}^S)^{1-\gamma}(\bar{L}^N+\bar{L}^S)^{\gamma}\left[\frac{\bar{M}^N}{\bar{M}^N+\bar{M}^S}(1-\gamma)+\frac{\bar{L}^N}{\bar{L}^N+\bar{L}^S}\gamma\right] \\
&\times \int_{z_{M,min}}^{z_{M,max}} e^{z_M^\beta m^{-1}(z_M)} \left(\frac{\phi_L(m^{-1}(z_M))}{\phi_M(z_M)}\frac{1}{m'(m^{-1}(z_M))}\right)^{\gamma}\phi_M(z_M)dz_M.
\end{aligned}$$

Using Theorem 7.6 **(Weighted AM-GM inequality)** in Cvetkovski (2012), for any $\gamma \in (0,1)$ and positive real numbers $\bar{M}^N$, $\bar{M}^S$, $\bar{L}^N$, and $\bar{L}^S$, the following inequality holds:

$$\frac{\bar{M}^N}{\bar{M}^N+\bar{M}^S}(1-\gamma)+\frac{\bar{L}^N}{\bar{L}^N+\bar{L}^S}\gamma \geq \left(\frac{\bar{M}^N}{\bar{M}^N+\bar{M}^S}\right)^{1-\gamma}\left(\frac{\bar{L}^N}{\bar{L}^N+\bar{L}^S}\right)^{\gamma}.$$

Equality occurs if and only if $\frac{\bar{M}^N}{\bar{L}^N}=\frac{\bar{M}^S}{\bar{L}^S}$. Therefore, total earnings in the North strictly increases from globalization. In the South, the argument is identical and, hence, it is omitted.

(iii) Because agents of the same skill within task are perfectly substitutable in globalization, they receive the same earnings.

(iv) The matching function $m(z_L)$ does not depend on the factor endowments $\bar{M}$ and $\bar{L}$ from equation (13). From equation (9), a one percent increase in $\frac{\bar{M}}{\bar{L}}$ raises $w(z_L)$ by $1-\gamma$ percent for all $z_L \in \mathcal{L}^W$. From equation (6), a one percent increase in $\frac{\bar{M}}{\bar{L}}$ reduces $r(z_M)$ by γ percent for all $z_M \in \mathcal{M}^W$. □

A.5 Proof of Lemma 1

Proof. (i) Suppose that there exists $z_L \in \mathcal{L}^N \cap \mathcal{L}^S$ such that $m^N(z_L) > m^S(z_L)$. Since $\frac{\phi_L^N(z_L')}{\phi_L^N(z_L)} \geq \frac{\phi_L^S(z_L')}{\phi_L^S(z_L)}$, we know that $\mathcal{L}^N \cap \mathcal{L}^S = [z_{L,min}^N, z_{L,max}^S]$. The positive assortative matching property of the matching function implies that $m^N(z_{L,min}^N) = z_{M,min} \leq m^S(z_{L,min}^N)$ and $m^S(z_{L,max}^S) = z_{M,max} \geq m^N(z_{L,max}^S)$. So there must exist $z_{L,min}^N \leq z_L^1 \leq z_L^2 \leq z_{L,max}^S$ and

$z_{M,min} \leq z_M^1 \leq z_M^2 \leq z_{M,max}$ such that

$$\begin{aligned} &\text{i)}\ \ m^N(z_L^1) = m^S(z_L^1) = z_M^1 \ \text{ and } \ m^N(z_L^2) = m^S(z_L^2) = z_M^2, \\ &\text{ii)}\ \ m^{N'}(z_L^1) \geq m^{S'}(z_L^1) \ \text{ and } \ m^{S'}(z_L^2) \geq m^{N'}(z_L^2), \\ &\text{iii)}\ \ m^N(z_L) > m^S(z_L) \ \text{ for all } \ z_L \in (z_L^1, z_L^2). \end{aligned}$$

$m^{N'}(z_L^1) \geq m^{S'}(z_L^1)$ and $m^{S'}(z_L^2) \geq m^{N'}(z_L^2)$ implies that:

$$\frac{m^{S'}(z_L^1)}{m^{S'}(z_L^2)} \leq \frac{m^{N'}(z_L^1)}{m^{N'}(z_L^2)}.$$

Using equation (8), we can derive the following inequality:

$$\frac{\phi_L^N(z_L^2)}{\phi_L^N(z_L^1)} \left[\frac{w^N(z_L^2)}{w^N(z_L^1)}\right]^{1/1-\gamma} \leq \frac{\phi_L^S(z_L^2)}{\phi_L^S(z_L^1)} \left[\frac{w^S(z_L^2)}{w^S(z_L^1)}\right]^{1/1-\gamma}.$$

$\frac{\phi_L^N(z_L^2)}{\phi_L^N(z_L^1)} \geq \frac{\phi_L^S(z_L^2)}{\phi_L^S(z_L^1)}$ requires that:

$$\frac{w^N(z_L^2)}{w^N(z_L^1)} \leq \frac{w^S(z_L^2)}{w^S(z_L^1)}.$$

However, this is a contradiction. Since $m^N(z_L) > m^S(z_L)$, it must be that

$$\frac{w^N(z_L^2)}{w^N(z_L^1)} > \frac{w^S(z_L^2)}{w^S(z_L^1)}.$$

Consequently, if $\frac{\phi_L^N(z_L')}{\phi_L^N(z_L)} \geq \frac{\phi_L^S(z_L')}{\phi_L^S(z_L)}$, then $m^N(z_L) \leq m^S(z_L)$ for all $\mathcal{L}^N \cap \mathcal{L}^S$.

(ii) First, let's prove that $\frac{\phi_L^N(z_L')}{\phi_L^N(z_L)} \geq \frac{\phi_L^S(z_L')}{\phi_L^S(z_L)}$ implies $\frac{\phi_L^N(z_L')}{\phi_L^N(z_L)} \geq \frac{\phi_L^W(z_L')}{\phi_L^W(z_L)}$. For any z_L' and z_L, $\frac{\phi_L^N(z_L')}{\phi_L^N(z_L)} \geq \frac{\phi_L^S(z_L')}{\phi_L^S(z_L)}$ implies that

$$\frac{\phi_L^N(z_L')}{\phi_L^N(z_L)} \geq \frac{\omega_L^N \phi_L^N(z_L') + \omega_L^S \phi_L^S(z_L')}{\omega_L^N \phi_L^N(z_L) + \omega_L^S \phi_L^S(z_L)}.$$

where $\omega_L^N \equiv \frac{\bar{L}^N}{\bar{L}^N + \bar{L}^S}$ and $\omega_L^S \equiv \frac{\bar{L}^S}{\bar{L}^N + \bar{L}^S}$. The right-hand side is $\frac{\phi_L^W(z_L')}{\phi_L^W(z_L)}$. Thus, $\frac{\phi_L^N(z_L')}{\phi_L^N(z_L)} \geq \frac{\phi_L^W(z_L')}{\phi_L^W(z_L)}$. Next, we prove that $\frac{\phi_L^N(z_L')}{\phi_L^N(z_L)} \geq \frac{\phi_L^S(z_L')}{\phi_L^S(z_L)}$ implies $\frac{\phi_L^W(z_L')}{\phi_L^W(z_L)} \geq \frac{\phi_L^S(z_L')}{\phi_L^S(z_L)}$. For any z_L' and

z_L, $\dfrac{\phi_L^N(z'_L)}{\phi_L^N(z_L)} \geq \dfrac{\phi_L^S(z'_L)}{\phi_L^S(z_L)}$ implies that

$$\frac{\omega_L^N \phi_L^N(z'_L) + \omega_L^S \phi_L^S(z'_L)}{\omega_L^N \phi_L^N(z_L) + \omega_L^S \phi_L^S(z_L)} \geq \frac{\phi_L^S(z'_L)}{\phi_L^S(z_L)}.$$

The left-hand side is $\dfrac{\phi_L^W(z'_L)}{\phi_L^W(z_L)}$. Thus, $\dfrac{\phi_L^W(z'_L)}{\phi_L^W(z_L)} \geq \dfrac{\phi_L^S(z'_L)}{\phi_L^S(z_L)}$. Therefore, $\dfrac{\phi_L^N(z'_L)}{\phi_L^N(z_L)} \geq \dfrac{\phi_L^S(z'_L)}{\phi_L^S(z_L)}$ implies $\dfrac{\phi_L^N(z'_L)}{\phi_L^N(z_L)} \geq \dfrac{\phi_L^W(z'_L)}{\phi_L^W(z_L)} \geq \dfrac{\phi_L^S(z'_L)}{\phi_L^S(z_L)}$. □

A.6 Proof of Proposition 5

Proof. (i) & (ii) Using Lemma 1, the proof is straightforward. The result also implies that $m^{N^{-1}}(z_M) \geq m^{W^{-1}}(z_M) \geq m^{S^{-1}}(z_M) \quad \forall z_M \in \mathcal{M}^W$.

(iii) & (iv) Define $\mathcal{I}_L^W = [z_{La}, z_{Lb}]$ as any connected subsets of $\mathcal{L}^W$ and $\mathcal{I}_M^W = [z_{Ma}, z_{Mb}]$ as any connected subsets of $\mathcal{M}^W$. By equations (10) and (12), the following world equilibrium conditions hold.

$$\ln w(z_{Lb'}) - \ln w(z_{La'}) = \int_{z_{La'}}^{z_{Lb'}} \frac{m(z)^\beta}{\gamma} dz, \quad \text{for all } z_{Lb'} > z_{La'} \text{ and } z_{La'}, z_{Lb'} \in \mathcal{I}_L^W,$$

$$\ln r(z_{Mb'}) - \ln r(z_{Ma'}) = \int_{z_{Ma'}}^{z_{Mb'}} \frac{\beta z^{\beta-1} m^{-1}(z)}{1-\gamma} dz, \quad \text{for all } z_{Mb'} > z_{Ma'} \text{ and } z_{Ma'}, z_{Mb'} \in \mathcal{I}_M^W.$$

Therefore, in the North, wage inequality widens as the matching function shifts upward while salary inequality narrows as the inverse matching function shifts downward. In the South, wage inequality narrows and salary inequality widens.

□

A.7 Proof of Lemma 3

Proof. (i) Suppose that there does not exist $z_L^* \in \mathcal{L}^N \cap \mathcal{L}^S$ such that $m^N(z_L) \geq m^S(z_L)$ for all $z_L \in [z_{L,min}^S, z_L^*]$ and $m^N(z_L) \leq m^S(z_L)$ for all $z_L \in [z_L^*, z_{L,max}^S]$. Since $\dfrac{\phi_L^N(z'_L)}{\phi_L^N(z_L)} \geq \dfrac{\phi_L^S(z'_L)}{\phi_L^S(z_L)}$ for all $z'_L \geq z_L \geq \hat{z}_L$, and $\dfrac{\phi_L^N(z'_L)}{\phi_L^N(z_L)} \leq \dfrac{\phi_L^S(z'_L)}{\phi_L^S(z_L)}$ for all $z_L \leq z'_L < \hat{z}_L$, we know that $\mathcal{L}^N \cap \mathcal{L}^S = [z_{L,min}^S, z_{L,max}^S]$. The positive assortative matching property of the matching function implies that $m^S(z_{L,min}^S) = z_{M,min} \leq m^N(z_{L,min}^S)$ and $m^S(z_{L,max}^S) = z_{M,max} \geq m^N(z_{L,max}^S)$. So there must exist $z_{L,min}^S \leq z_L^0 < z_L^1 < z_L^2 \leq z_{L,max}^S$ and $z_{M,min} \leq z_M^0 < z_M^1 < z_M^2 \leq z_{M,max}$

such that

i) $m^N(z_L^0) = m^S(z_L^0) = z_M^0$, $m^N(z_L^1) = m^S(z_L^1) = z_M^1$ and $m^N(z_L^2) = m^S(z_L^2) = z_M^2$,

ii) $m^{N'}(z_L^0) \leq m^{S'}(z_L^0)$, $m^{N'}(z_L^1) \geq m^{S'}(z_L^1)$ and $m^{N'}(z_L^2) \leq m^{S'}(z_L^2)$,

iii) $m^N(z_L) < m^S(z_L)$ for all $z_L \in (z_L^0, z_L^1)$ and $m^N(z_L) > m^S(z_L)$ for all $z_L \in (z_L^1, z_L^2)$.

There are two possible cases: a) $z_L^1 < \hat{z}_L$ and b) $z_L^1 \geq \hat{z}_L$. In case a), $m^{N'}(z_L^0) \leq m^{S'}(z_L^0)$ and $m^{N'}(z_L^1) \geq m^{S'}(z_L^1)$ implies that:

$$\frac{m^{N'}(z_L^1)}{m^{N'}(z_L^0)} \geq \frac{m^{S'}(z_L^1)}{m^{S'}(z_L^0)}.$$

Using equation (8), we can derive the following inequality:

$$\frac{\phi_L^N(z_L^1)}{\phi_L^N(z_L^0)} \left[\frac{w^N(z_L^1)}{w^N(z_L^0)}\right]^{1/1-\gamma} \geq \frac{\phi_L^S(z_L^1)}{\phi_L^S(z_L^0)} \left[\frac{w^S(z_L^1)}{w^S(z_L^0)}\right]^{1/1-\gamma}.$$

$\frac{\phi_L^N(z_L^1)}{\phi_L^N(z_L^0)} \leq \frac{\phi_L^S(z_L^1)}{\phi_L^S(z_L^0)}$ requires that:

$$\frac{w^N(z_L^1)}{w^N(z_L^0)} \geq \frac{w^S(z_L^1)}{w^S(z_L^0)}.$$

However, this is a contradiction. Since $m^N(z_L) < m^S(z_L)$ for all $z_L \in (z_L^0, z_L^1)$, it must be that

$$\frac{w^N(z_L^1)}{w^N(z_L^0)} \leq \frac{w^S(z_L^1)}{w^S(z_L^0)}.$$

In case b), the argument is identical and, hence, it is omitted. □

A.8 Proof of Proposition 9

Proof. The matching function $m(z_L)$ does not depend on technology level α from equation (13), which implies that the matching function $m(z_L)$ does not change. From equation (9) and equation (11), a one percent increase in α raises both $r(z_M)$ and $w(z_L)$ by one percent for all $z_M \in \mathcal{M}$ and $z_L \in \mathcal{L}$. □

A.9 Proof of Proposition 10

Proof. Since γ governs the share of output that goes to workers and managers, we can easily show that the workers' share increases and the managers' share declines due to an increase in the parameter γ.

Next, we show that if $\gamma^N > \gamma^S$, then $m^N(z_L) \geq m^S(z_L)$ for all $\mathcal{L}^N \cap \mathcal{L}^S$. Suppose that there exists $z_L \in \mathcal{L}^N \cap \mathcal{L}^S$ such that $m^N(z_L) < m^S(z_L)$. Since $\phi_L^N(z_L) = \phi_L^S(z_L)$, we know that $\mathcal{L}^N \cap \mathcal{L}^S = [z_{L,min}^N, z_{L,max}^N] = [z_{L,min}^S, z_{L,max}^S]$. There must exist $z_{L,min} \leq z_L^1 \leq z_L^2 \leq z_{L,max}$ and $z_{M,min} \leq z_M^1 \leq z_M^2 \leq z_{M,max}$ such that

$$\begin{aligned} &\text{i)}\ \ m^N(z_L^1) = m^S(z_L^1) = z_M^1 \ \text{ and } \ m^N(z_L^2) = m^S(z_L^2) = z_M^2, \\ &\text{ii)}\ \ m^{N'}(z_L^1) \leq m^{S'}(z_L^1) \ \text{ and } \ m^{S'}(z_L^2) \leq m^{N'}(z_L^2), \\ &\text{iii)}\ \ m^N(z_L) < m^S(z_L) \ \text{ for all } \ z_L \in (z_L^1, z_L^2). \end{aligned}$$

$m^{N'}(z_L^1) \leq m^{S'}(z_L^1)$ and $m^{S'}(z_L^2) \leq m^{N'}(z_L^2)$ implies that:

$$\frac{m^{S'}(z_L^1)}{m^{S'}(z_L^2)} \geq \frac{m^{N'}(z_L^1)}{m^{N'}(z_L^2)}.$$

Using equation (8), we can derive the following inequality:

$$\left[\frac{w^S(z_L^1)}{w^S(z_L^2)}\right]^{1/1-\gamma^S} \geq \left[\frac{w^N(z_L^1)}{w^N(z_L^2)}\right]^{1/1-\gamma^N}.$$

$\gamma^N > \gamma^S$ requires that:

$$\frac{w^N(z_L^2)}{w^N(z_L^1)} \geq \frac{w^S(z_L^2)}{w^S(z_L^1)}.$$

However, this is a contradiction. Since $\gamma^N > \gamma^S$ and $m^N(z_L) < m^S(z_L)$, it must be that

$$\frac{w^N(z_L^2)}{w^N(z_L^1)} < \frac{w^S(z_L^2)}{w^S(z_L^1)}.$$

Consequently, if $\gamma^N > \gamma^S$, then $m^N(z_L) \geq m^S(z_L)$ for all $\mathcal{L}^N \cap \mathcal{L}^S$.

Lastly, the size of the most skilled firms in the North is the inverse of $\dfrac{m^{N'}(z_{L,max})\bar{M}^N\phi_M^N(m^N(z_{L,max}))}{\bar{L}^N\phi_L^N(z_{L,max})}$ and the size of the most skilled firms in the South is the inverse of $\dfrac{m^{S'}(z_{L,max})\bar{M}^S\phi_M^S(m^S(z_{L,max}))}{\bar{L}^S\phi_L^S(z_{L,max})}$. Since $m^N(z_L) \geq m^S(z_L)$ for all $\mathcal{L}^N \cap \mathcal{L}^S$ and $m^N(z_{L,max}) = m^S(z_{L,max})$, it must be that $m^{N'}(z_{L,max}) \leq m^{S'}(z_{L,max})$. Hence, the size of the most skilled firms (weakly) increases in the South. The proof is identical in the case of the size of the least skilled firms and, hence, it is omitted. □

A.10 Proof of Proposition 11

Proof. Suppose that there exists $z_L \in \mathcal{L}^N \cap \mathcal{L}^S$ such that $m^N(z_L) < m^S(z_L)$. Since $\phi_L^N(z_L) = \phi_L^S(z_L)$, we know that $\mathcal{L}^N \cap \mathcal{L}^S = [z_{L,min}^N, z_{L,max}^N] = [z_{L,min}^S, z_{L,max}^S]$. There must exist $z_{L,min} \leq z_L^1 \leq z_L^2 \leq z_{L,max}$ and $z_{M,min} \leq z_M^1 \leq z_M^2 \leq z_{M,max}$ such that

$$
\begin{aligned}
&\text{i)}\;\; m^N(z_L^1) = m^S(z_L^1) = z_M^1 \;\text{ and }\; m^N(z_L^2) = m^S(z_L^2) = z_M^2,\\
&\text{ii)}\;\; m^{N'}(z_L^1) \leq m^{S'}(z_L^1) \;\text{ and }\; m^{S'}(z_L^2) \leq m^{N'}(z_L^2),\\
&\text{iii)}\;\; m^N(z_L) < m^S(z_L) \;\text{ for all }\; z_L \in (z_L^1, z_L^2).
\end{aligned}
$$

$m^{N'}(z_L^1) \leq m^{S'}(z_L^1)$ and $m^{S'}(z_L^2) \leq m^{N'}(z_L^2)$ implies that:

$$\frac{m^{S'}(z_L^1)}{m^{S'}(z_L^2)} \geq \frac{m^{N'}(z_L^1)}{m^{N'}(z_L^2)}.$$

Using equation (8), we can derive the following inequality:

$$\left[\frac{e^{m(z_L^2)^{\beta^S} z_L^2}}{e^{m(z_L^1)^{\beta^S} z_L^1}}\right]\left[\frac{w^S(z_L^1)}{w^S(z_L^2)}\right] \geq \left[\frac{e^{m(z_L^2)^{\beta^N} z_L^2}}{e^{m(z_L^1)^{\beta^N} z_L^1}}\right]\left[\frac{w^N(z_L^1)}{w^N(z_L^2)}\right].$$

Given that $z_{L,min}^N = z_{L,min}^S \geq 1$ and $z_{M,min}^N = z_{M,min}^S \geq 1$, $\beta^N > \beta^S$ implies $\left[\frac{e^{m(z_L^2)^{\beta^S} z_L^2}}{e^{m(z_L^1)^{\beta^S} z_L^1}}\right] \leq \left[\frac{e^{m(z_L^2)^{\beta^N} z_L^2}}{e^{m(z_L^1)^{\beta^N} z_L^1}}\right]$. This, in turn, requires that:

$$\frac{w^N(z_L^2)}{w^N(z_L^1)} \geq \frac{w^S(z_L^2)}{w^S(z_L^1)}.$$

However, this is a contradiction. Since $\beta^N > \beta^S$ and $m^N(z_L) < m^S(z_L)$, it must be that

$$\frac{w^N(z_L^2)}{w^N(z_L^1)} < \frac{w^S(z_L^2)}{w^S(z_L^1)}.$$

Consequently, if $\beta^N > \beta^S$, then $m^N(z_L) \geq m^S(z_L)$ for all $\mathcal{L}^N \cap \mathcal{L}^S$. In equation (10), since $\beta^N > \beta^S$ and $m^N(z_L) \geq m^S(z_L)$ for all $\mathcal{L}^N \cap \mathcal{L}^S$, wage inequality rises. From equation (9), we can easily show that $w^N(z_{L,min}) > w^S(z_{L,min})$ using $\beta^N > \beta^S$ and $m^{N'}(z_{L,min}) > m^{S'}(z_{L,min})$. From equation (11), since $\beta^N > \beta^S$ and $m^{N'}(m^{-1}(z_{M,max})) < m^{S'}(m^{-1}(z_{M,max}))$, we can show that $r^N(z_{M,max}) > r^S(z_{M,max})$. Therefore, in the South, the wage schedule shifts upward, wage inequality rises, and the highest skilled manager's salary rises due to the manager-biased technical change. □

A.11 Proof of Lemma 5

Proof. Suppose $z_*^N < z_*^S$. Since $\frac{\phi^N(z')}{\phi^N(z)} \geq \frac{\phi^S(z')}{\phi^S(z)}$, we know that $\mathcal{Z}^N \cap \mathcal{Z}^S = [z_{min}^N, z_{max}^S]$. The positive assortative matching property of the matching function implies that $m^N(z_{min}^N) = z_*^N \leq m^S(z_{min}^N)$ and $m^S(z_*^N) \leq m^N(z_*^N) = z_{max}^N$. So there must exist $z_{min}^N < z^1 < z_*^N$ such that $m^N(z^1) = m^S(z^1) = z^2$ and $m^{N'}(z^1) > m^{S'}(z^1)$. $\frac{\phi^N(z^2)}{\phi^N(z^1)} \geq \frac{\phi^S(z^2)}{\phi^S(z^1)}$ requires that $w^N(z^1) > w^S(z^1)$. Because the number of workers per manager is $N = \left[\frac{\gamma\alpha e^{m(z)^\beta z}}{w(z)}\right]^{1/1-\gamma}$ and the salary schedule is given by $r(z) = (1-\gamma)\,\alpha e^{z^\beta m^{-1}(z)} N^\gamma$, it must be that $r^N(z^2) < r^S(z^2)$. However, this is a contradiction. Since $m^N(z^1) = m^S(z^1)$, $m^N(z_*^N) > m^S(z_*^N)$, and $w^N(z^1) > w^S(z^1)$, it must be that $w^N(z_*^N) > w^S(z_*^N)$. By Assumption 1 and $w^N(z_*^N) = r^N(z_*^N) > w^S(z_*^N)$, it must be that $r^N(z_*^S) > r^S(z_*^S)$. Note that $m^{N-1}(z_*^S) > m^{S-1}(z_*^S)$ and $m^{N-1}(z^2) > m^{S-1}(z^2) = z^1$. Hence, it must be that $r^N(z^2) > r^S(z^2)$. Consequently, if $\frac{\phi^N(z')}{\phi^N(z)} \geq \frac{\phi^S(z')}{\phi^S(z)} \quad \forall z' \geq z$, then, $z_*^N \geq z_*^S$. □

A.12 Proof of Proposition 12

Proof. (i) Lemma 5 and the the fact that $\phi^W(z)$ satisfies $\frac{\phi^N(z')}{\phi^N(z)} \geq \frac{\phi^W(z')}{\phi^W(z)} \geq \frac{\phi^S(z')}{\phi^S(z)}$ prove the proposition.

(ii) The positive assortative matching property of the matching function implies that $m^{W^{-1}}(z_{N,max}) \leq m^{N^{-1}}(z_{N,max})$ and $m^{W^{-1}}(z_*^N) \geq m^{N^{-1}}(z_*^N)$. Hence, there must exist a cutoff skill level $\psi \in [z_*^N, z_{N,max}]$ such that $m^{W^{-1}}(\psi) = m^{N^{-1}}(\psi)$.

(iii) The positive assortative matching property of the matching function implies that $m^W(z_{S,min}) \geq m^S(z_{S,min})$ and $m^W(z_*^S) \leq m^S(z_*^S)$. Hence, there must exist a cutoff skill level $\zeta \in [z_{S,min}, z_*^S]$ such that $m^W(\zeta) = m^S(\zeta)$.

(iv) The relation between the matching function and the inequality is proved in the Proof of Proposition 5. From Assumption 1, when workers become managers within any connected subset of agents, inequality rises within the interval.

(v) The proof is identical to the proof in (iv) and, hence, it is omitted. □

A.13 Proof of Proposition 13

Proof. (i) Lemma 5 and the the fact that $\phi^W(z)$ satisfies $\frac{\phi^N(z')}{\phi^N(z)} \geq \frac{\phi^W(z')}{\phi^W(z)} \geq \frac{\phi^S(z')}{\phi^S(z)}$ prove the proposition.

(ii) The positive assortative matching property of the matching function implies that $m^W(z_{min}) \leq m^N(z_{min})$ and $m^W(z_*^W) = z_{max} \geq m^N(z_*^W)$. Hence, there must exist a cutoff skill level $\psi \in [z_{min}, z_*^N]$ such that $m^W(\psi) = m^N(\psi)$. The positive assortative matching property of the matching function implies that $m^{W^{-1}}(z_*^N) = z_{min} \geq m^{N^{-1}}(z_*^N)$ and $m^{W^{-1}}(z_{max}) \leq m^{N^{-1}}(z_{max})$. Hence, there must exist a cutoff skill level $\psi \in [z_*^N, z_{max}]$ such that $m^{W^{-1}}(\psi) = m^{N^{-1}}(\psi)$.

(iii) The proof is identical to the proof in (ii) and, hence, it is omitted.

(iv) The relation between the matching function and the inequality is proved in the Proof of Proposition 5. From Assumption 1, when workers become managers within any connected subset of agents, inequality rises within the interval.

(v) The proof is identical to the proof in (iv) and, hence, it is omitted. □

A.14 Proof of Proposition 16

Proof. (i) Lemma 5 and the the fact that $\phi^W(z)$ satisfies $\frac{\phi^N(z')}{\phi^N(z)} \geq \frac{\phi^W(z')}{\phi^W(z)} \geq \frac{\phi^S(z')}{\phi^S(z)}$ prove the proposition.

(ii) & (iii) Suppose that $m^O(z) = m^S(z) \quad \forall z \in [z_{S,min}, z_{N,min}]$ and $m^O(z) = m^N(z) \quad \forall z \in [z_{N,min}, z_*^N]$. We must show that $m^O(z) \leq m^W(z) \quad \forall z \in [z_{S,min}, z_*^W]$. Suppose not. Then, there must exist $z_{S,min} \leq z^1 \leq z^2 \leq z_*^W$ such that

$$
\begin{aligned}
\text{i)}\ & m^O(z^1) = m^W(z^1) \ \text{ and } \ m^O(z^2) = m^W(z^2),\\
\text{ii)}\ & m^{O'}(z^1) \geq m^{W'}(z^1) \ \text{ and } \ m^{O'}(z^2) \leq m^{W'}(z^2),\\
\text{iii)}\ & m^O(z) > m^N(z) \ \text{ for all } \ z \in (z^1, z^2).
\end{aligned}
$$

$m^{O'}(z^1) \geq m^{W'}(z^1)$ and $m^{O'}(z^2) \leq m^{W'}(z^2)$ implies that:

$$\frac{m^{O'}(z^2)}{m^{O'}(z^1)} \leq \frac{m^{W'}(z^2)}{m^{W'}(z^1)}.$$

We can derive the following inequality:

$$\frac{\phi^O(z^2)}{\phi^O(z^1)} \left[\frac{w^O(z^2)}{w^O(z^1)}\right]^{1/1-\gamma} \leq \frac{\phi^W(z^2)}{\phi^W(z^1)} \left[\frac{w^W(z^2)}{w^W(z^1)}\right]^{1/1-\gamma}.$$

$\frac{\phi^O(z^2)}{\phi^O(z^1)} \geq \frac{\phi^W(z^2)}{\phi^W(z^1)}$ requires that:

$$\frac{w^O(z^2)}{w^O(z^1)} \leq \frac{w^W(z^2)}{w^W(z^1)}.$$

However, this is a contradiction. Since $m^O(z) > m^W(z)$, it must be that

$$\frac{w^O(z^2)}{w^O(z^1)} > \frac{w^W(z^2)}{w^W(z^1)}.$$

Consequently, we have $m^O(z) \leq m^W(z) \quad \forall z \in [z_{S,min}, z_*^W]$.

(iv) The relation between the matching function and the inequality is proved in the Proof of Proposition 5. From Assumption 1, when workers become managers within any connected subset of agents, inequality rises within the interval.

(v) The proof is identical to the proof in (iv) and, hence, it is omitted. □

A.15 Proof of Example 1

Proof. Suppose that $(z_M^*)^N = (z_M^*)^S$, $(z_L^*)^N = (z_L^*)^S$, $m_1^N(z_L) = m_1^S(z_L)$ for all $z_L \in [z_L^*, z_{L,max}]$, and $m_2^N(z_L) = m_2^S(z_L)$ for all $z_L \in [z_{L,min}, z_L^*]$. Since $m_1^N(z_L) = m_1^S(z_L)$ for all $z_L \in [z_L^*, z_{L,max}]$, there must exist $z_L^* \leq z_L^2 \leq z_L^1 \leq z_{L,max}$ and $z_M^* \leq z_M^2 \leq z_M^1 \leq z_{M,max}$ such that $m_1^{N'}(z_L^1) = m_1^{S'}(z_L^1)$ and $m_1^{N'}(z_L^2) = m_1^{S'}(z_L^2)$, which implies that:

$$\frac{m_1^{S'}(z_L^1)}{m_1^{S'}(z_L^2)} = \frac{m_1^{N'}(z_L^1)}{m_1^{N'}(z_L^2)}.$$

We can derive the following equality:

$$\frac{\phi_L^S(z_L^1)}{\phi_L^S(z_L^2)} \left[\frac{w_1^S(z_L^1)}{w_1^S(z_L^2)}\right]^{1/1-\gamma_1} = \frac{\phi_L^N(z_L^1)}{\phi_L^N(z_L^2)} \left[\frac{w_1^N(z_L^1)}{w_1^N(z_L^2)}\right]^{1/1-\gamma_1}.$$

$\frac{\phi_L^N(z_L^1)}{\phi_L^N(z_L^2)} > \frac{\phi_L^S(z_L^1)}{\phi_L^S(z_L^2)}$ requires that:

$$\frac{w_1^S(z_L^1)}{w_1^S(z_L^2)} < \frac{w_1^N(z_L^1)}{w_1^N(z_L^2)}.$$

However, this is a contradiction.Since $m_1^N(z_L) = m_1^S(z_L)$ for all $z_L \in [z_L^*, z_{L,max}]$, it must be that

$$\frac{w_1^S(z_L^1)}{w_1^S(z_L^2)} = \frac{w_1^N(z_L^1)}{w_1^N(z_L^2)}.$$

$\square$

A.16 Proof of Example 2

Proof. Suppose that $(\widetilde{z})^N = (\widetilde{z})^S$, $(\overline{z})^N = (\overline{z})^S$ $(z^*)^N = (z^*)^S$, $m_1^N(z) = m_1^S(z)$ for all $z \in [z^*, \overline{z}]$, and $m_2^N(z) = m_2^S(z)$ for all $z \in [z_{min}, z^*]$. Since $m_1^N(z) = m_1^S(z)$ for all $z \in [z^*, \overline{z}]$, there must exist $z^* < z^1 < \overline{z}$ such that $m_1^N(z^1) = m_1^S(z^1) = z^2$ and $m_1^{N'}(z^1) = m_1^{S'}(z^1)$. $\dfrac{\phi^N(z^2)}{\phi^N(z^1)} > \dfrac{\phi^S(z^2)}{\phi^S(z^1)}$ requires that $w_1^N(z^1) > w_1^S(z^1)$. Because the number of workers per manager is $N = \left[\dfrac{\gamma\alpha e^{m(z)^\beta z}}{w(z)}\right]^{1/1-\gamma}$ and the salary schedule is given by $r(z) = (1-\gamma)\,\alpha e^{z^\beta m^{-1}(z)} N^\gamma$, it must be that $r_1^N(z^2) < r_1^S(z^2)$. However, this is a contradiction. $\square$

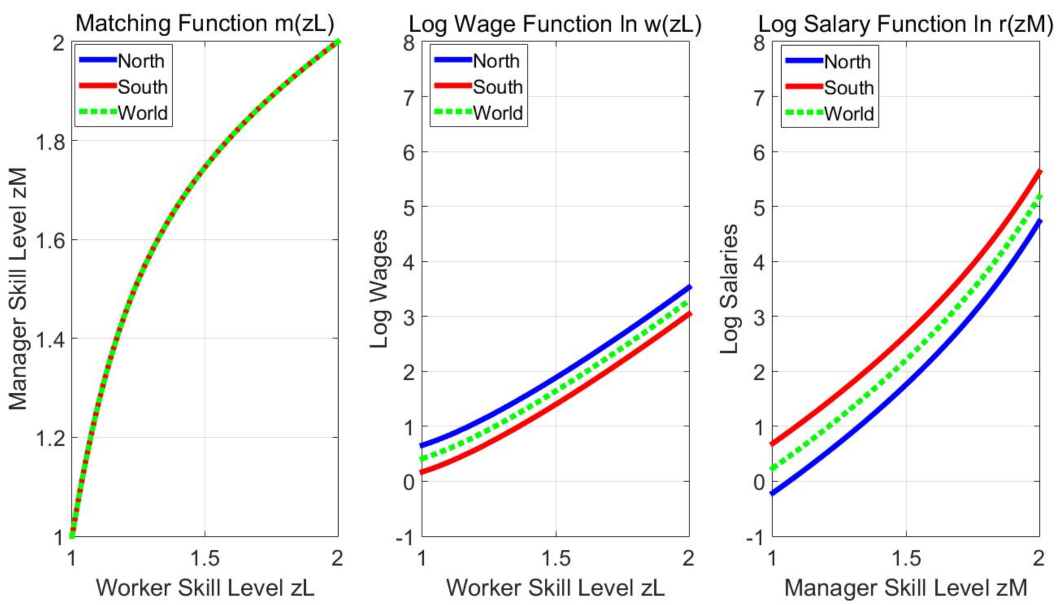
B Numerical Exercises

B.1 Numerical Exercises for Proposition 4

Table 5: Sets of parameter values for Proposition 4

Parameter	Description	North	South	World
$\bar{M}$	Number of managers	**200**	**100**	**300**
$\bar{L}$	Number of workers	**500**	**1,000**	**1,500**
$\mathcal{M}$	Set of manager skill levels	[1,2]	[1,2]	[1,2]
$\mathcal{L}$	Set of worker skill levels	[1,2]	[1,2]	[1,2]
k_M	Shape parameter for manager skill distribution	2	2	2
k_L	Shape parameter for worker skill distribution	2	2	2
β	Sensitivity to manager skill level	1.2	1.2	1.2
γ	Span of control	0.65	0.65	0.65

Figure 9: The impact of offshoring under cross-country differences in endowments

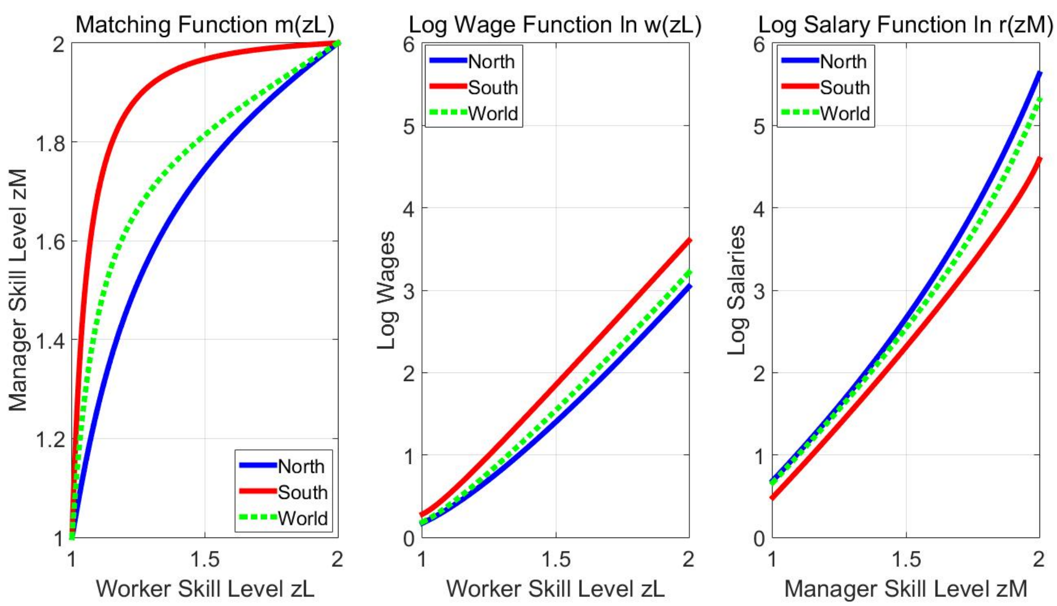


B.2 Numerical Exercises for Proposition 5

Table 6: Sets of parameter values for Proposition 5

Parameter	Description	North	South	World
$\bar{M}$	Number of managers	100	100	200
$\bar{L}$	Number of workers	1,000	1,000	2,000
$\mathcal{M}$	Set of manager skill levels	[1,2]	[1,2]	[1,2]
$\mathcal{L}$	Set of worker skill levels	[1,2]	[1,2]	[1,2]
k_M	Shape parameter for manager skill distribution	2	2	2
k_L	Shape parameter for worker skill distribution	**2**	**10**	$\mathbf{k_L \in (2, 10)}$
β	Sensitivity to manager skill level	1.2	1.2	1.2
γ	Span of control	0.65	0.65	0.65

Figure 10: The impact of offshoring under cross-country differences in worker distribution

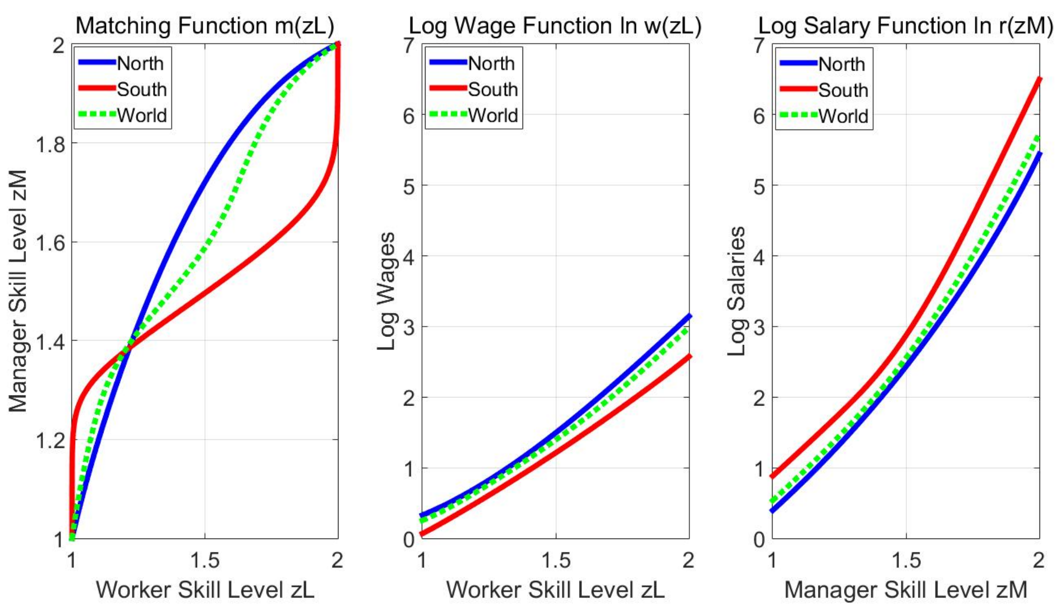


B.3 Numerical Exercises for Proposition 8

Table 7: Sets of parameter values for Proposition 8

Parameter	Description	North	South	World
$\bar{M}$	Number of managers	100	100	200
$\bar{L}$	Number of workers	1,000	1,000	2,000
$\mathcal{M}$	Set of manager skill levels	[1,2]	[1,2]	[1,2]
$\mathcal{L}$	Set of worker skill levels	[1,2]	[1,2]	[1,2]
μ_M	Mean parameter for manager skill distribution	1.5	1.5	1.5
μ_L	Mean parameter for worker skill distribution	1.5	1.5	1.5
σ_M	Variance parameter for manager skill distribution	**0.6**	**0.1**	$\sigma_{\mathbf{M}} \in (\mathbf{0.1}, \mathbf{0.6})$
σ_L	Variance parameter for worker skill distribution	0.3	0.3	0.3
β	Sensitivity to manager skill level	1.2	1.2	1.2
γ	Span of control	0.65	0.65	0.65

Figure 11: The impact of offshoring under cross-country differences in manager distribution

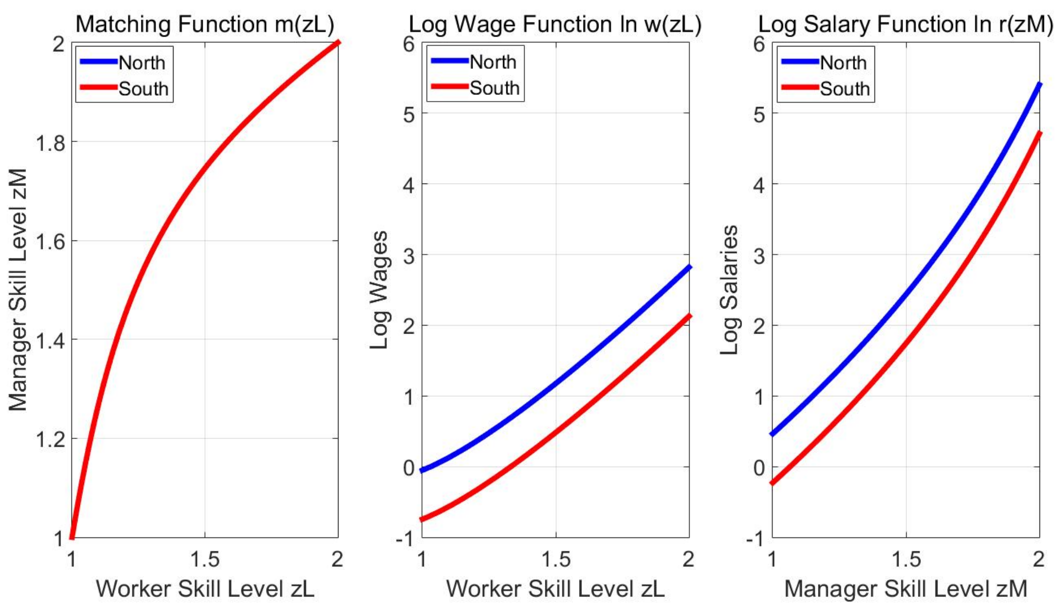


B.4 Numerical Exercises for Proposition 9

Table 8: Sets of parameter values for Proposition 9

Parameter	Description	Value (North)	Value (South)
$\bar{M}$	Number of managers	100	100
$\bar{L}$	Number of workers	1,000	1,000
$\mathcal{M}$	Set of manager skill levels	[1,2]	[1,2]
$\mathcal{L}$	Set of worker skill levels	[1,2]	[1,2]
k_M	Shape parameter for manager skill distribution	2	2
k_L	Shape parameter for worker skill distribution	2	2
α	Hicks-neutral technology	**0.8**	**0.4**
β	Sensitivity to manager skill level	1.2	1.2
γ	Span of control	0.65	0.65

Figure 12: The distributional impact of Hicks-neutral technology transfer



C Many-to-Many Matching

Let us study a case where a firm hiring *some number of managers* M of the same skill level z_M paired up with *some number of workers* L of the same skill level z_L can produce a final good Y. The production function in the economy is defined as follows:

$$Y = F(z_M, z_L, M, L) = \alpha e^{z_M^\beta z_L} M^{1-\gamma} L^\gamma, \quad \alpha \in \{0, 1\}, \quad \beta > 1, \quad 0 < \gamma < 1.$$

Consider a firm hires M number of managers of the same skill z_M. Given the output price $P_Y = 1$, the wage schedule $w(z_L)$, and the salary schedule $r(z_M)$, the firm chooses the skill level of its workers z_L and the number of workers L to maximize profits:

$$\pi(z_L, L; z_M, M) = \alpha e^{z_M^\beta z_L} M^{1-\gamma} L^\gamma - w(z_L)L - r(z_M)M.$$

Differentiating with respect to L yields the conditional workers per manager

$$N(z_L; z_M) = \left[\frac{\gamma \alpha e^{z_M^\beta z_L}}{w(z_L)}\right]^{1/1-\gamma}.$$

where $N = \frac{L}{M}$. The expression is identical to that of the many-to-one matching case, and we find the same equilibrium condition after solving a firm's profit maximization problem.